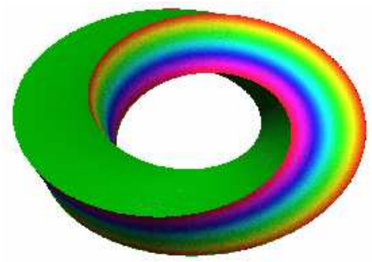
Universität Bielefeld
Fakultät für Mathematik



Third Workshop on Random Dynamical Systems

18 – 20 November 2009

Department of Mathematics
University of Bielefeld
Lecture Hall H10

This workshop is part of the conference program of the DFG-funded CRC 701
Spectral Structures and Topological Methods in Mathematics
at the University of Bielefeld

Organizers: Wolf-Jürgen Beyn and Barbara Gentz

http://www.math.uni-bielefeld.de/~gentz/pages/WS09/RDS09/RDS09.html

Programme

Wednesday, 18 November 2009

9:00 – 9:30	*Registration and coffee*
9:30 – 9:35	*Welcome*
9:35 – 10:20	**Omar Lakkis** (University of Sussex) Some computational aspects of stochastic phase-field models
10:35 – 11:00	*Coffee break*
11:00 – 11:45	**Raphael Kruse** (Universität Bielefeld) A stability concept for stochastic onestep and multistep methods
12:00 – 14:30	*Lunch break*
14:30 – 15:15	**Michael Allman** (Warwick Mathematics Institute) Breaking the chain and the effect of mass
15:30 – 16:00	*Coffee break*
16:00 – 16:45	**Erika Hausenblas** (Universität Salzburg) SPDEs driven by Levy processes
17:00 – 17:45	**Peter Kotelenez** (Case Western Reserve University, Cleveland, OH) Stochastic flows and signed measure valued stochastic partial differential equations

Thursday, 19 November 2009

9:00 – 9:30	*Registration and coffee*
9:30 – 10:15	**Andreas Rößler** (TU Darmstadt) Improved Multi-Level Monte Carlo Methods for SDEs
10:30 – 11:00	*Coffee break*
11:00 – 11:45	**Arnulf Jentzen** (Universität Bielefeld) Convergence of the stochastic Euler scheme for locally Lipschitz coefficients
12:00 – 14:30	*Lunch break*
14:30 – 15:15	**Peter Reimann** (Universität Bielefeld) Suppression of thermally activated escape by heating
15:30 – 16:00	*Coffee break*
16:00 – 16:45	**Nils Berglund** (MAPMO–CNRS, Orléans) Metastablility for Ginzburg–Landau-type SPDEs with space–time white noise
17:00 – 17:45	**Yuri Bakhtin** (Georgia Tech, Atlanta, GA) Small noise asymptotics for noisy heteroclinic networks
18:30 –	*Joint dinner at Bültmannshof* Bültmannshof, Kurt-Schumacher-Straße 17a, 33615 Bielefeld (Please note: For the dinner, prior registration is required.)

Friday, 20 November 2009

9:00 – 9:30	*Coffee*
9:30 – 10:15	**Evelyn Buckwar** (Heriot-Watt University, Edinburgh) Linear stability analysis for stochastic Theta-methods applied to systems of SODEs
10:30 – 11:00	*Coffee break*
11:00 – 11:45	**Peter Kloeden** (Goethe-Universität Frankfurt a. M.) Spatial Discretization of Dynamical Systems
12:00 – 14:00	*Lunch*
14:00 – 14:45	**Mihály Kovács** (University of Otago, Dunedin, New Zealand) Finite element approximation of the stochastic wave equation
15:00 – 15:45	**Dirk Blömker** (Universität Augsburg) Local shape of random invariant manifolds
16:00	*Closing of the workshop and coffee*

Abstracts

Michael Allman (Warwick Mathematics Institute)

Breaking the chain and the effect of mass

We consider the motion of a Brownian particle in a time-dependent potential well undergoing a symmetric pitchfork bifurcation. In the absence of noise, the particle falls into the right-hand well due to an additional drift term. We investigate how much noise is needed to give an equal probability of falling into either well in the limit of small noise and whether there is a difference between the overdamped and underdamped cases. This is related to the behaviour of a chain of three particles being stretched slowly.

Yuri Bakhtin (Georgia Institute of Technology, Atlanta, GA)

Small noise asymptotics for noisy heteroclinic networks

I will start with the deterministic dynamics generated by a vector field that has several unstable critical points connected by heteroclinic orbits. A perturbation of this system by white noise will be considered. I will study the limit of the resulting stochastic system in distribution (under appropriate time rescaling) as the noise intensity vanishes. It is possible to describe the limiting process in detail, and, in particular, interesting non-Markov effects arise. There are situations where this result provides more precise exit asymptotics than the classical Wentzell–Freidlin theory.

Nils Berglund (MAPMO–CNRS, Orléans)

Metastablility for Ginzburg–Landau-type SPDEs with space–time white noise

We consider one-dimensional PDEs of Ginzburg–Landau type on a finite interval $[0, L]$, with either periodic or Neumann boundary conditions. These equations admit two stable, stationary solutions, which are spatially homogeneous, and, depending on L, one or several unstable stationary solutions. When weak space–time white noise is added to the system, the stable solutions become metastable, and rare transitions between them take place. In order to determine the rate of these transitions, we have to extend the classical Eyring–Kramers law to infinite-dimensional situations, and include degenerate saddle points in the energy functional. Our approach extends results obtained by Bovier, Eckhoff, Gayrard and Klein for the finite-dimensional, nondegenerate case, and is based on potential-theoretic tools.

Joint work with Florent Barret (Ecole Polytechnique) and Barbara Gentz (Universität Bielefeld).

References:

– N.B. and Barbara Gentz, Anomalous behavior of the Kramers rate at bifurcations in classical field theories, J. Phys. A: Math. Theor. 42:052001 (2009)

– N.B. and Barbara Gentz, The Eyring–Kramers law for potentials with non-quadratic saddles, Preprint arXiv/0807.1681

Dirk Blömker (Universität Augsburg)

Local shape of random invariant manifolds

We consider an SPDE of Burgers type with simple multiplicative noise. Near a change of stability, we investigate the local shape of the random invariant manifold around the deterministic fixed-point. This approach is compared to the approximation of SPDEs via amplitude equations.
Joint work with Wei Wang (Adelaide/Nanjing).

Evelyn Buckwar (Heriot-Watt University, Edinburgh)

Linear stability analysis for stochastic Theta-methods applied to systems of SODEs

An important issue arising in the analysis of numerical methods for approximating the solution of a differential equation is concerned with the ability of the methods to preserve the asymptotic properties of equilibria. For stochastic ordinary differential equations investigations in this direction have mainly focussed on scalar equations so far. In this talk I will first examine the structure of stochastic perturbations that are known to a.s. stabilise or destabilise the equilibrium solutions of systems of differential equations. These perturbation structures, encoded in the diffusion coefficient matrix of a linear system, will provide the basis for choosing test equations. Then I will present a mean-square and a.s. stability analysis of the Theta-discretisations of these test equations. The talk is based on joint work with Conall Kelly and Thorsten Sickenberger.

Erika Hausenblas (Universität Salzburg)

SPDEs driven by Levy processes

Since I am actually working with Poisson random measure I will first introduce Poisson random measures and its relation to Levy processes. Then I will introduce the stochastic integration on Banach spaces with respect to Poisson random measures. In the next part some existence and uniqueness results of SPDEs will be presented. In particular, existence and uniqueness of SPDEs with respcet to space time Levy noise will be presented. In the third part nonlinear SPDEs, resp. SPDEs with only continuous coefficients will be tackled. Here, first, the terminus of a martingale solution will be defined. Then, some existence results will be given.

Arnulf Jentzen (Universität Bielefeld)

Convergence of the stochastic Euler scheme for locally Lipschitz coefficients

Stochastic differential equations are often simulated with the Monte Carlo Euler method. Convergence of this method is well understood in the case of globally Lipschitz continuous coefficients of the stochastic differential equation. The important case of superlinearly growing coefficients, however, remained an open problem for a long time now. The main difficulty is that numerically weak convergence fails to hold in many cases of superlinearly growing coefficients. In this talk we overcome this difficulty and establish convergence of the Monte Carlo Euler method for a large class of one-dimensional stochastic differential equations whose drift functions have at most polynomial growth.

Peter Kloeden (Goethe-Universität Frankfurt a. M.)

Spatial Discretization of Dynamical Systems

The effects of round-off error can have a profound effect on dynamical behaviour when a dynamical system, here generated by a difference equation, are simulated in a computer. In particular, chaotic dynamics may collapse onto trivial steady state behaviour or spurious cycles may arise. Invariant measures are robuster to such spatial discretization of a dynamical system. Their approximation using permutations and Markov chains is reviewed here.

Peter Kotelenez (Case Western Reserve University, Cleveland, OH)

Stochastic flows and signed measure valued stochastic partial differential equations

We derive a class of quasi-linear stochastic partial differential equations (SPDEs) from the empirical distributions of stochastic flows of systems of stochastic ordinary differential equations (SODEs) on $\mathbb{R}^d$. We show that the solutions of the SODEs generate a.s. a homeomorphism from the initial conditions onto the solutions at time t. Generalizing results of Kotelenez (2008), Ch. 8,[1] from positive measure valued SPDEs to signed measure valued SPDEs, the solutions of the SPDEs may be represented as follows:

$$\mathcal{X}(t) = \mathcal{X}^{+}(t) - \mathcal{X}^{-}(t), \qquad (*)$$

where $\mathcal{X}^{+}(t)$ and $\mathcal{X}^{-}(t)$ are positive measures with the following flow representation

$$\mathcal{X}^{\pm}(t) = \int_{\mathbb{R}^d} \delta_{(\bar{r}(t,\mathcal{X},q))} \mathcal{X}^{\pm}(0, dq). \qquad (**)$$

$\mathcal{X}(0) = \mathcal{X}^{+}(0) - \mathcal{X}^{-}(0)$ is the initial distribution and $\mathcal{X}^{\pm}(0)$ is the Hahn-Jordan decomposition of $\mathcal{X}(0)$. Further, $\bar{r}(t, \mathcal{X}, q)$ is the flow of solutions of the SODEs, depending on the "empirical distribution" $\mathcal{X}(\cdot)$ and the initial condition q. The flow properties imply that $\mathcal{X}^{\pm}(t)$ is the Hahn-Jordan decomposition of $\mathcal{X}(t)$. Smoothness and uniqueness hold for smooth initial conditions and smooth coefficients of the SODEs. This result has numerous applications in $2D$ fluid mechanics and other areas.

[1] Cf. Kotelenez, P. (2008) *Stochastic Ordinary and Stochastic Partial Differential Equations: Transition from Microscopic to Macroscopic Equations*, Springer-Verlag, Berlin-Heidelberg-New York.

Mihály Kovács (University of Otago, Dunedin, New Zealand)

Finite element approximation of the stochastic wave equation

Semidiscrete finite element approximation of the linear stochastic wave equation with additive noise is studied in a semigroup framework. Optimal error estimates for the deterministic problem are obtained under minimal regularity assumptions. These are used to prove strong and weak convergence estimates for the stochastic problem. The rate of weak convergence is found to be twice that of strong convergence under essentially the same regularity assumptions on the covariance operator of the Wiener process. The theory presented here applies to multi-dimensional domains and spatially correlated noise. Numerical examples illustrate the theory. This talk is based on the preprints:

– Finite element approximation of the linear stochastic wave equation with additive noise (with S. Larsson, and F. Saedpanah). Preprint 2009:18, Department of Mathematical Sciences, Chalmers University of Technology.
– Weak convergence of finite element approximations of linear stochastic evolution equations with additive noise (with S. Larsson, and F. Lindgren). Preprint 2009:37, Department of Mathematical Sciences, Chalmers University of Technology.

Raphael Kruse (Universität Bielefeld)

A stability concept for stochastic onestep and multistep methods

There already exists a well-established convergence theory concerning onestep and multistep methods for stochastic ordinary differential equations. It is also known that the zero-stability of a stochastic multistep method is characterized by Dahlquist's root condition. In this talk we show that all of these results can be embedded into one unifying theory. This theory is based on the standard framework of consistency, stability and convergence as it has been formulated in abstract terms by F. Stummel in the theory of discrete approximations. Moreover, a special choice of function spaces and norms, namely a stochastic version of Spijker's norm, and a strong version of Dahlquist's root condition allow us to prove bistability. From this property we derive two-sided estimates of the strong error which can be used to prove optimal rates of convergence for Itô-Taylor schemes and BDF methods.

Omar Lakkis (University of Sussex)

Some computational aspects of stochastic phase-field models

Our main goal is the (correct) numerical approximation of stochastic phase-fields such as, the Allen-Cahn problem with additive white noise in one-dimensional space:

$$\partial_t u(x,t) - \Delta u(x,t) + \frac{1}{\epsilon^2}(u(x,t)^3 - u(x,t)) = \epsilon^\gamma \partial_{xt} W(x,t), \quad x \in (-1,1) \text{ and } t \in (0,T],$$

with Neumann boundary conditions and $u(x,0) = u_0(x)$. We then have to relate our numerical results to theoretical results from the probabilistic analysis of scaling limits conducted mainly by Funaki and Brassesco, De Masi & Presutti in the 1990's [2, 1].

From a computational view-point, the main difficulty for a rigorous numerical discretization of this SPDE, which is the baby-version of more complicated phase-field systems, is the presence of the time-space white noise as a forcing term [6, 7, 5].

We conduct a finite element discretization in two stages: (1) we regularize the white noise and study the regularized problem, and (2) we approximate the regularized problem and derive a finite element Monte Carlo simulation scheme [3].

In this talk, I will review recent advances on the topic where we analyze the interface behavior, as its thickness ϵ stays finite. Asymptotic analysis shows that the stochastic solution and its approximation remain "close" to a set of functions where the interface makes sense. This is particularly useful to relate numerical results to theory and it has the potential to define stochastic diffuse interfaces in dimensions higher than one [4].

(*Based on joint work with Markos Katsoulakis, Giorgos Kossioris and Marco Romito*)

References

1. S. Brassesco, A. De Masi, and E. Presutti, *Brownian fluctuations of the interface in the $D = 1$ Ginzburg-Landau equation with noise*, Ann. Inst. H. Poincaré Probab. Statist. **31** (1995), no. 1, 81–118. MR 96f:82029
2. Tadahisa Funaki, *The scaling limit for a stochastic PDE and the separation of phases*, Probab. Theory Relat. Fields **102** (1995), 221–288.
3. Markos Katsoulakis, Giorgos Kossioris, and Omar Lakkis, *Noise regularization and computations for the 1-dimensional stochastic Allen-Cahn problem*, Interfaces and Free Boundaries **9** (2007), no. 1, 1–30.
4. Georgios Kossioris, Omar Lakkis, and Marco Romito, *Interface dynamics for the stochastic Allen–Cahn model: asymptotics and computations*, Tech. report, In preparation 2009.
5. Di Liu, *Convergence of the spectral method for stochastic Ginzburg-Landau equation driven by space-time white noise*, Commun. Math. Sci. **1** (2003), no. 2, 361–375. MR MR1980481 (2004c:60187)
6. Grant Lythe, *Stochastic PDEs: domain formation in dynamic transitions*, Proceedings of the VIII Reunión de Física Estadística, FISES '97, vol. 4, Anales de Física, Monografías RSEF, 1998, pp. 55—63.
7. Tony Shardlow, *Stochastic perturbations of the Allen-Cahn equation*, Electron. J. Differential Equations (2000), No. 47, 19 pp. (electronic). MR 2001k:74082

Peter Reimann (Universität Bielefeld)

Suppression of thermally activated escape by heating

The problem of thermally activated escape over a potential barrier is solved by means of path integrals for one-dimensional reaction dynamics with very general time dependences. For a suitably chosen but still quite simple static potential landscape, the net escape rate may be substantially reduced by temporally increasing the temperature above its unperturbed constant level.

Andreas Rößler (TU Darmstadt)

Improved Multi-Level Monte Carlo Methods for SDEs

We consider the problem of weak approximation of solutions of stochastic differential equations (SDEs). Therefore, the multi-level Monte Carlo method is applied together with a second order weak approximation method. While the weak first order Euler-Maruyama scheme is applied for the reduction of variance, we propose to apply some higher order method in order to minimize the bias. Then, the mean-square error of the estimator for the expectation of a functional applied to the solution of the underlying SDE has asynptotically nearly optimal order. As the main novelty, the computational effort can be reduced by a factor 1/4 if a second order weak approximation scheme is applied. This will be revealed by some numerical examples.

Registered participants

Michael Allman	(Warwick Mathematics Institute)
Yuri Bakhtin	(Georgia Institute of Technology, Atlanta, GA)
Nils Berglund	(MAPMO–CNRS, Orléans)
Wolf-Jürgen Beyn	(Universität Bielefeld)
Dirk Blömker	(Universität Augsburg)
Evelyn Buckwar	(Heriot-Watt University, Edinburgh)
Dongxi Li	(Universität Bielefeld)
Etienne Emmrich	(Universität Bielefeld)
Uta Freiberg	(Universität Siegen)
Barbara Gentz	(Universität Bielefeld)
Erika Hausenblas	(Universität Salzburg)
Daniel Henkel	(TU Darmstadt)
Thorsten Hüls	(Universität Bielefeld)
Arnulf Jentzen	(Universität Bielefeld)
Jens Kemper	(Universität Bielefeld)
Peter Kloeden	(Goethe-Universität Frankfurt a.M.)
Peter Kotelenez	(Case Western Reserve University, Cleveland, OH)
Mihály Kovács	(University of Otago, Dunedin, New Zealand)
Raphael Kruse	(Universität Bielefeld)
Omar Lakkis	(University of Sussex)
Wei Liu	(Universität Bielefeld)
Denny Otten	(Universität Bielefeld)
Peter Reimann	(Universität Bielefeld)
Marina Reizakis	(Springer-Verlag)
Andreas Rößler	(TU Darmstadt)
Marco Romito	(Università di Firenze)
Jens Rottmann-Matthes	(Universität Bielefeld)
Sebastian Schmitz	(Universität Bielefeld)
Felipe Torres Tapia	(Universität Bielefeld)
Matthias Weber	(HTW Dresden)

(as of 13 November 2009)