Universität Bielefeld
Fakultät für Mathematik

SFB 701

Fifth Workshop on Random Dynamical Systems

4 – 5 October 2012

Department of Mathematics
University of Bielefeld
Room V2–210/216

This workshop is part of the conference program of the DFG-funded CRC 701
Spectral Structures and Topological Methods in Mathematics
at the University of Bielefeld

Organizers: Barbara Gentz (Bielefeld) and Maria G. Westdickenberg (Aachen)

`http://www.math.uni-bielefeld.de/~gentz/pages/WS12/RDS12/RDS12.html`

Programme

Thursday, 4 October 2012

8:30 – 9:00	*Registration and coffee*
9:00 – 9:05	*Welcome*
9:05 – 9:45	**Etienne Pardoux** (Aix-Marseille Université) How noise might select an invariant measure of a dynamical system. A toy example
9:55 – 10:35	**Michael Scheutzow** (TU Berlin) Completeness and semiflows for stochastic differential equations with monotone drift
Coffee break	
11:10 – 11:50	**Stefan Neukamm** (MPI für Mathematik in den Naturwissenschaften, Leipzig) Quantitative estimates in stochastic homogenization
12:00 – 12:15	**Olga Chugreeva** (RWTH Aachen) Ginzburg–Landau energy for stochastically perturbed equation
Lunch break	
14:00 – 14:40	**Moritz Kaßmann** (Universität Bielefeld) Local regularity of nonlocal operators
14:50 – 15:05	**Lisa Beck** (Universität Bonn) Pathwise regularity of solutions to some spdes
15:10 – 15:25	**Konrad Klepel** (Universität Augsburg) Amplitude equation for the generalized Swift Hohenberg equation with noise
Coffee break	
16:00 – 16:40	**Max von Renesse** (Universität Leipzig) Ergodic properties of stochastic curve shortening flows
16:50 – 17:30	**Ilya Pavlyukevich** (Friedrich-Schiller-Universität Jena) Small noise asymptotics of integrated Ornstein–Uhlenbeck processes driven by α-stable Lévy processes
19:30 –	*Joint dinner in the city centre* fabrikart, Münzstraße 5, 33602 Bielefeld, phone +49 521 9630 8408 (Please note: For the dinner, prior registration is required.)

Friday, 5 October 2012

8:45 – 9:00	*Coffee*
9:00 – 9:40	**Elisabetta Scoppola** (University of Roma Tre) Asymptotically exponential hitting times and metastability: a pathwise approach
9:50 – 10:30	**Nils Berglund** (Université d'Orléans) Some applications of quasistationary distributions to random Poincaré maps
Coffee break	
11:10 – 11:50	**Eric Vanden-Eijnden** (Courant Institute, NYU) Reaction pathways of metastable Markov chains with application to Lennard-Jones clusters reorganization
12:00 – 12:15	**Simon Weber** (University of Warwick) The sharp interface limit of the stochastic Allen–Cahn equation in one space-dimension
Lunch break	
14:00 – 14:40	**Alessandra Bianchi** (Università di Padova) Quasi-stationary measures and metastability
14:50 – 15:05	**André Schlichting** (Universität Bonn) The Eyring–Kramers formula for Poincaré and logarithmic Sobolev inequalities
Coffee break	
15:40 – 16:20	**Hendrik Weber** (University of Warwick) Invariant measure of the stochastic Allen–Cahn equation: the regime of small noise and large system size
16:30 – 17:00	**Leonid Koralov** (University of Maryland, College Park, MD) Random and deterministic perturbations of dynamical systems

Abstracts

Lisa Beck (Universität Bonn)

Pathwise regularity of solutions to some spdes

In the deterministic theory there are both classes of systems of partial differential equations, where all solutions are actually more regular, and examples of systems which admit solutions which start from a regular initial condition and develop a discontinuity in finite time. In this talk we discuss some of these aspects under the effect of random perturbations. More precisely, we give one example of how pde methods can be used to deduce pathwise regularity properties of solutions. Secondly, we address the possibility that noise might even prevent the emergence of singularities in cases where the underlying deterministic system has irregular solutions (similar phenomena are known in connection to stability and well-posedness for some classes of systems).

Joint project with F. Flandoli, Pisa.

Nils Berglund (Université d'Orléans)

Some applications of quasistationary distributions to random Poincaré maps

We are interested in the long-time behaviour of continuous-space, discrete-time substochastic Markov chains. Our main motivation for considering such processes is that they describe, via "random Poincaré maps", the dynamics of stochastic differential equations depending periodically on one variable. We will present ways to approximate the principal eigenvalue and the spectral gap of these Markov chains, as well as the associated eigenfunctions. As a first application, we show that the distribution of first-exit locations of a planar stochastic differential equation from the interior of an unstable periodic orbit is governed by a periodicized Gumbel distribution. As a second application, we will derive some properties of interspike interval distributions for the FitzHugh–Nagumo model, describing the membrane potential of a neuron.

Joint work with Barbara Gentz (Bielefeld) and Damien Landon (Dijon).

Alessandra Bianchi (Università di Padova)

Quasi-stationary measures and metastability

In this talk I will present some recent results, obtained in collaboration with A. Gaudilliere, concerning the characterization of metastability in the sense of Lebowitz and Penrose for Markov processes on finite configuration space and in some asymptotic regime. By comparison between the restricted ensemble and the quasi-stationary measure and introducing "soft measures" as interpolation between them, we will derive simple and practical hypotheses to establish metastability.

In particular, by means of potential theoretic tools, we will provide sharp estimates on mean exit time and transition time, and prove their asymptotic exponential laws. Related estimates on the relaxation and mixing time, derived via two-sided variational principles, will be also discussed.

Olga Chugreeva (RWTH Aachen)

Ginzburg–Landau energy for stochastically perturbed equation

The Ginzburg-Landau equations are evolution equations related to the Ginzburg-Landau energy functional

$$E_\varepsilon(u) := \int_D \frac{1}{2}|\nabla u|^2 + \frac{1}{4\varepsilon^2}\left(1 - |u|^2\right)^2.$$

If the boundary condition with nontrivial winding number is posed, the solutions are forced to present vortices. It is possible to derive an ODE ruling their motion provided the growth of $E_\varepsilon(u)$ is controlled. When a random forcing of a certain intensity is introduced, this general picture is preserved, but the classical machinery is no more relevant.

In this talk I discuss the behavior of the energy functional for the solutions of a Ginzburg-Landau equation with multiplicative noise. The Itô's lemma delivers an explicit equation for the energy dynamics, which in its turn enables one to conclude on the properties of the Jacobian, a quantity essential for tracking down vortex paths.

Moritz Kaßmann (Universität Bielefeld)

Local regularity of nonlocal operators

In recent years several important regularity results for differential operators of second order have been extended or rather transferred to integrodifferential operators of order $\alpha/2 \in (0,1)$. We review some of these works with a special emphasis on two issues: the notion of ellipticity for nonlocal operators and robustness of the results as $\alpha \to 2-$. Under comparability assumptions on the energies we establish a parabolic Harnack inequality which is robust.

The talk is based on recent joint works with Dyda, Felsinger, Mimica and Rang.

Konrad Klepel (Universität Augsburg)

Amplitude equation for the generalized Swift Hohenberg equation with noise

We derive an amplitude equation for a stochastic partial differential equation (SPDE) of Swift–Hohenberg type on a bounded domain. We consider a nonlinearity that is composed of a stable cubic and an unstable quadratic term, and assume that the noise acts only on the constant mode, shaking the whole system at once.

Due to the natural separation of timescales, solutions can be approximated by the slow dominant modes. Nevertheless, via the nonlinearity, the noise gets transmitted to these modes. Thus the amplitude equation, describing the complex-valued amplitude of the dominant Fourier modes, contains multiplicative noise, as well as additional deterministic terms arising from the noise. This may lead to stabilization effects with increasing noise strength. The extension of the result to unbounded domains is work in progress and discussed briefly.

Joint work with Dirk Blömker.

Leonid Koralov (University of Maryland, College Park, MD)

Random and deterministic perturbations of dynamical systems

The talk is based on the recent work, some of which is joint with D. Dolgopyat and some joint with M. Freidlin. First, I'll talk about deterministic and stochastic perturbations of incompressible flows. Even in the case of purely deterministic perturbations, the long-time behavior of such systems can be stochastic, in a certain sense. The proper understanding of the limiting behavior of deterministically perturbed flows requires regularization by a small random term, but turns out not to depend on the particular form of the regularization.

I'll also discuss the extension of the Freidlin and Wentzell theory of large deviations and of their results on averaging to the case when the generator of the process is perturbed by a small non-linear term.

Stefan Neukamm (MPI für Mathematik in den Naturwissenschaften, Leipzig)

Quantitative estimates in stochastic homogenization

We consider the diffusion equation $-\nabla \cdot (a(x)\nabla u) = f$ on the lattice $\mathbb{Z}^d$ with random coeffients $a(x)$. For stationary and ergodic coefficients $a(x)$, it is known from the theory of stochastic homogenization that the macroscopic behavior of the equation is captured by the homogenized equation $-\nabla \cdot a_{\text{hom}}\nabla u$ where a_{hom} is deterministic and homogeneous in space. The *homogenized coefficients* a_{hom} are given by a homogenization formula that involves the solution to the *corrector* problem $-\nabla \cdot (a(x)(\xi + \nabla\phi) = 0$ on $\mathbb{Z}^d$.

In practice the homogenization formula has to be approximated, since (I) the corrector problem can only be solved for a small number of realizations of the ensemble $\langle\cdot\rangle$, and (II) the corrector problem can only be solved in a finite domain of large diameter L.

In this talk we present quantitative estimates on the corrector equation and the associated diffusion semigroup that rely on a spectral gap for a Glauber dynamics on coefficient fields and decay estimates on the gradient of the parabolic, non-constant coefficient Greens function. As an application we obtain optimal error estimate for the approximation of a_{hom} by the periodization method.

This is joint work with Antoine Gloria, INRIA Lille, and Felix Otto, MPI Leipzig.

Etienne Pardoux (Aix-Marseille Université)

How noise might select an invariant measure of a dynamical system. A toy example

Our work is motivated by the following open problem (as well as by other similar problems). Consider a $2D$ Navier–Stokes equation (say on the $2D$ torus $\mathbb{T}$ for simplicity) with additive white noise of the form

$$\dot{u} - \varepsilon\Delta u + (u \cdot \nabla)u + \nabla p = \sqrt{\varepsilon}\dot{W}, \quad \operatorname{div}(u) = 0,$$

where W is an $L^2(\mathbb{R}^2)$–valued Brownian motion which is such that for each $\varepsilon > 0$, the above SPDE has a unique invariant measure μ_ε (see Hairer, Mattingly [1] for the best known conditions under which this is true). Kuksin [2] has shown that the collection $\{\mu_\varepsilon,\, \varepsilon > 0\}$ is tight, and that any limit of a converging subsequence is an invariant measure of the Euler equation.

The remaining open problem is whether or not the whole sequence converges, and if yes, towards which particular invariant measure of the Euler equation?

We do not claim to solve this difficult *true problem*. Rather, we consider a much simpler problem, namely a $3D$ SDE with damping of the order of ε and additive white noise multiplied by $\sqrt{\varepsilon}$. Our very simple *toy problem* has however in common with the *true problem* the fact that the limiting deterministic undamped ODE possesses conserved quantities and countably many invariant measures.

We will describe the solution of our *toy problem*, where the sequence of invariant measures of the stochastic systems converges to a certain mixture of invariant measures of the limiting ODE.

This is joint work with Jonathan Mattingly (Duke Univ.).

References

[1] J. Mattingly, M. Hairer, Ergodicity of the $2D$ Navier–Stokes equation, *Ann. of Math.* **164**, 2006

[2] S.B. Kuksin, On Distribution of Energy and Vorticity for Solutions of 2D Navier–Stokes Equation with Small Viscosity, *Comm. Math. Phys.* **284**, 2008.

Ilya Pavlyukevich (Friedrich-Schiller-Universität Jena)

Small noise asymptotics of integrated Ornstein–Uhlenbeck processes driven by α-stable Lévy processes

We study the behaviour of a one-dimensional integrated Ornstein–Uhlenbeck process driven by an α-stable Lévy process of small amplitude, $\alpha \in (0, 2)$. We show that the continuous integrated Ornstein–Uhlenbeck process converges to the driving pure jump α-stable Lévy process in the Skorokhod M_1-topology. In particular this allows us to determine the limiting distribution of its first passage times.

This is a joint work with R. Hintze (FSU Jena).

Max von Renesse (Universität Leipzig)

Ergodic properties of stochastic curve shortening flows

We discuss $1+1$ dimensional random interface models known as stochastic curve shortening flow. Well-posedness is established in a variational SPDE framework. For the long time behaviour we prove ergodicity using the lower-bound-technique by Peszat/Szarek/Komorowski. Finally we show polynomial stabilty using a-priori estimates on the invariant measure.

Michael Scheutzow (TU Berlin)

Completeness and semiflows for stochastic differential equations with monotone drift

We consider stochastic differential equations on a Euclidean space driven by a Kunita-type semimartingale field satisfying a one-sided local Lipschitz condition. We address questions of local and global existence and uniqueness of solutions as well as existence of a local or global semiflow. Further, we will provide sufficient conditions for strong p-completeness, i.e. almost sure non-explosion for subsets of dimension p under the local solution semiflow.

Part of the talk is based on joint work with Susanne Schulze and other parts with Xue-Mei Li (Warwick).

André Schlichting (Universität Bonn)

The Eyring–Kramers formula for Poincaré and logarithmic Sobolev inequalities

We consider a diffusion process on a potential landscape which is given by a smooth Hamiltonian function in the regime of small noise. We give a new proof of the Eyring–Kramers formula for the Poincaré inequality of the associated generator of the diffusion and use this new approach to obain also a sharp estimate of the constant of the logarithmic Sobolev inequality. The proof consists of a divide-and-conquer strategy, which mimics the two-scale approach introduced by Grunewald, Otto, Villani, and Westdickenberg. The Eyring–Kramers formula follows as a simple corollary from two main ingredients: The first one shows that the Gibbs measure restricted to a basin of attraction has a "good"Poincaré constant providing the fast convergence of the diffusion to metastable states. The second ingredient is a refinement of the mean-difference estimate introduced by Chafai and Malrieu. There, we propose a weighted transportation distance, which contains the main contribution to the Poincaré and logarithmic Sobolev constant, resulting from exponential long waiting times of jumps between metastable states of the diffusion.

(joint work with Georg Menz)

Elisabetta Scoppola (University of Roma Tre)

Asymptotically exponential hitting times and metastability: a pathwise approach

Metastability is strictly related to hitting times to rare events. A short review on the topic is presented and some results, in preparation, on the exponential behavior in the general non reversible case are also discussed.

Eric Vanden-Eijnden (Courant Institute, NYU)

Reaction pathways of metastable Markov chains with application to Lennard-Jones clusters reorganization

Lennard-Jones particles tend to aggregate in tetrahedra that can then form larger clusters with icosahedral symmetry. For clusters with certain number of particles (e.g. 38), the icosahedral structure is not the one with lowest energy, which is an FCC-based structure with octahedral symmetry. We may ask: How does the system reorganize itself after self-assembly to reach this ground state of its energy and how long does this process take? The dynamics of this reorganization can be modeled by Markov chain, i.e. a random walk on a network whose nodes are the local energy minima of the cluster and whose edges are the minimum energy paths connecting these minima with weights that depend on the energy barrier to be crossed to hop from one local minima to one of its neighbor on the network. It is natural to think about using large deviation theory (LDT) to understand the pathways of reorganization on such a network. Here, however, we show that the predictions of LDT are only valid at very low temperature when the time-scale of reorganization is extremely large. At more moderate temperatures, the system remains highly metastable (in the sense that there exists low lying eigenvalues in the spectrum of the chain) but reorganization occurs by a pathway that is different than that predicted by LDT and can be quantified precisely using different tools. These results are applicable to other Markov chains displaying metastability over two or more states.

Hendrik Weber (University of Warwick)

Invariant measure of the stochastic Allen-Cahn equation: the regime of small noise and large system size

We study the invariant measure of the one-dimensional stochastic Allen–Cahn equation for a small noise strength and a large but finite system. We endow the system with inhomogeneous Dirichlet boundary conditions that enforce at least one transition from -1 to 1. We are interested in the competition between the "energy" that should be minimized due to the small noise strength and the "entropy" that is induced by the large system size.

Our methods handle system sizes that are exponential with respect to the inverse noise strength, up to the "critical" exponential size predicted by the heuristics. We capture the competition between energy and entropy through upper and lower bounds on the probability of extra transitions between +1 and -1. These bounds are sharp on the exponential scale and imply in particular that the probability of having one and only one transition from -1 to +1 is exponentially close to one. In addition, we show that the position of the transition layer is uniformly distributed over the system on scales larger than the logarithm of the inverse noise strength.

Our arguments rely on local large deviation bounds, the strong Markov property, the symmetry of the potential, and measure-preserving reflections.

Simon Weber (University of Warwick)

The sharp interface limit of the stochastic Allen–Cahn equation in one space-dimension

The Allen–Cahn equation is a reaction–diffusion equation used in a phenomenological model of phase separation; due to numerical simulations it is a well-known fact that perturbing the equation with space–time noise gives much better approximations of the actual phenomenon. In particular the motion of the phase boundaries in the sharp interface limit, corresponding to cooling down a material to absolute zero is an interesting problem that has been studied in the stochastic case since the mid-nineties. Working in one space-dimension, we extend known results of the case of one interface to the more general case of finitely many interfaces on an interval. On the right time-scale, this converges to annihilating brownian motions in the sharp interface limit.

This is joint work with Martin Hairer.

Registered participants

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(as of 1 October 2012)