Eighth Workshop on Random Dynamical Systems

5 – 7 November 2015

Department of Mathematics
University of Bielefeld
Room V2–210/216

This workshop is part of the conference programme of the DFG-funded CRC 701
Spectral Structures and Topological Methods in Mathematics
at the Faculty of Mathematics at the University of Bielefeld

Organizers: Barbara Gentz (Bielefeld) and Peter Imkeller (Berlin)

http://www.math.uni-bielefeld.de/~gentz/pages/WS15/RDS15/RDS15.htm

Programme

Thursday, 5 November 2015

8:30 – 9:00	*Registration and coffee in V3–201*
9:00 – 9:10	**Friedrich Götze** (Speaker of the CRC 701) *Opening of the workshop*
9:10 – 9:55	**Robert C. Dalang** (Ecole Polytechnique Fédérale de Lausanne) Polarity of points for systems of linear stochastic pde's in critical dimensions
10:05 – 10:50	**Nils Berglund** (Université d'Orléans) Regularity structures and renormalisation of FitzHugh–Nagumo SPDEs in three space dimensions
11:00 – 11:20	*Coffee in V3–201*
11:20 – 12:05	**Jeroen Lamb** (Imperial College London) On the problem of pitchfork bifurcation with additive noise
12:15 – 12:40	**Sébastien Dutercq** (Université d'Orléans) Interface dynamics of a metastable mass-conserving diffusion
12:45 – 14:15	*Lunch*
14:15 – 15:00	**Julian Newman** (Imperial College London) *Conditions for synchronisation in random dynamical systems*
15:10 – 15:35	**Jan Gairing** (Humboldt-Universität zu Berlin) Exponential growth rates and ergodic theory for jump diffusions
15:40 – 16:25	**Maria G. Westdickenberg** (RWTH Aachen) Local minima and energy barriers in the d-dimensional Cahn–Hilliard energy landscape
16:30 – 17:15	*Tea in V3–201*
17:15 – 18:15	*Mathematical Colloquium of the Faculty of Mathematics* **Michael Loss** (Georgia Institute of Technology, Atlanta, GA) Optimal functional inequalities and flows

All talks will take place in V2–210/216.

Friday, 6 November 2015

8:45 – 9:00	*Coffee in V3–201*
9:00 – 9:45	**Volker Betz** (TU Darmstadt) Spatial random permutations – old and new results
9:55 – 10:40	**Luigi Amedeo Bianchi** (Universität Augsburg) Pattern size in Gaussian fields from spinodal decomposition
10:50 – 11:20	*Coffee in V3–201*
11:20 – 12:05	**Giambattista Giacomin** (Université Paris Diderot) Weak noise and non hyperbolic unstable fixed points
12:15 – 12:40	**Alexandra Neamtu** (Friedrich-Schiller-Universität Jena) Dynamics of non-densely defined stochastic evolution equations
12:45 – 14:15	*Lunch*
14:15 – 15:00	**Christopher J. Bose** (University of Victoria, BC) Limit theorems for random intermittent maps
15:10 – 15:55	**Paul Razafimandimby** (Montanuniversität Leoben) Some results on the stochastic Langrangian averaged Euler equations on non-smooth domain: Well-posedness and regularity
16:05 – 16:30	*Coffee in V3–201*
16:30 – 16:55	**Manon Baudel** (Université d'Orléans) Spectral theory for random Poincaré maps
17:00 – 17:45	**Martina Hofmanová** (TU Berlin) Stochastic mean curvature flow *— cancelled —*
19:00 –	*Joint dinner at Wernings Weinstube in the city centre* Alter Markt 1, 33602 Bielefeld, ☎ +49 521 136 51 51 `http://www.wernings-weinstube.de` Please note: For the dinner, prior registration is required.

All talks will take place in V2–210/216.

Saturday, 7 November 2015

8:45 – 9:00	*Coffee in V3–201*
9:00 – 9:45	**Jiří Černý** (Universität Wien) Aging of the Metropolis dynamics of the REM
9:55 – 10:40	**Francesca R. Nardi** (Eindhoven University of Technology) Asymptotic behavior of hitting times for Metropolis Markov chains and applications to the hard-core model on grids
10:50 – 11:10	*Coffee in V3–201*
11:10 – 11:55	**Eva Löcherbach** (Université de Cergy-Pontoise) On oscillating systems of interacting Hawkes processes
12:05 – 12:50	**Nicolas Perkowski** (Humboldt-Universität zu Berlin) *Paracontrolled KPZ equation*
13:00 – 14:30	*Lunch at the Westend-Cafeteria (next to the indoor pool)*

All talks will take place in V2–210/216.

Abstracts

Manon Baudel (Université d'Orléans)

Spectral theory for random Poincaré maps

We consider an ordinary differential equation with N stable periodic orbits perturbed by weak noise, for which we want to quantify the rare transitions between periodic orbits. The behaviour of the stochastic system is analysed through the spectrum of the kernel of a Markov process describing successive returns to a Poincaré section. Because of the N stable periodic orbits, N eigenvalues of the Markov process are expected to be close to 1.

Using Laplace transforms of first-hitting times and a Dirichlet boundary value problem, we show that the eigenvalue problem on the Poincaré section can be reduced to an eigenvalue problem on a union of small balls surrounding the stable orbits. In this way we obtain a homogeneous integral equation, whose eigenvalues are the roots of a Fredholm determinant and are directly linked to the eigenvalues of our Markov process.

Using perturbative arguments and Harnack inequalities, we approximate the Markov process by an N-state Markov chain. Under some assumptions, we can prove that the N eigenvalues of the discrete-space Markov chain are close to 1.

This is a work in progress with Nils Berglund.

Nils Berglund (Université d'Orléans)

Regularity structures and renormalisation of FitzHugh–Nagumo SPDEs in three space dimensions

Martin Hairer's recent theory of regularity structures allows to define a notion of solution for very singular stochastic PDEs, including (but not limited to) a large class of suitably renormalised parabolic SPDEs driven by space-time white noise. After providing an introduction to regularity structures, we will present an extension of the theory to systems of parabolic SPDEs coupled to an ODE, where the unknown is a function of time and two- or three-dimensional space. These equations include in particular a FitzHugh–Nagumo system describing the evolution of action potentials of a large population of neurons, as well as models with multidimensional gating variables. Our main result shows local existence of solutions to a renormalised version of the equations, with explicit expressions for the renormalisation constants.

Joint work with Christian Kuehn (TU Vienna).

Volker Betz (TU Darmstadt)

Spatial random permutations – old and new results

I will introduce the model of spatial random permutations, and explain how it is connected to the theory of Bose–Einstein condensation. Then I will discuss the few things that are known about this model, as well as some conjectures suggested by numerical simulation. Finally I will present important open problems, and very recent progress made jointly with Lorenzo Taggi (TU Darmstadt).

Luigi Amedeo Bianchi (Universität Augsburg)

Pattern size in Gaussian fields from spinodal decomposition

We study the two-dimensional snake-like pattern that arises in phase separation of alloys described by spinodal decomposition in the Cahn?Hilliard?Cook model. These are somewhat universal patterns due to an overlay of eigenfunctions of the Laplacian with a similar wave-number. Similar structures appear in other models like reaction-diffusion systems describing animal coats? patterns or vegetation patterns in desertification.

Our main result studies random functions given by cosine Fourier series with independent Gaussian coefficients, that dominate the dynamics in the Cahn?Hilliard model. This is not a cosine process, as the sum is taken over domains in Fourier space that not only grow and scale with a parameter of order $1/\varepsilon$, but also move to infinity. Moreover, the model under consideration is neither stationary nor isotropic.

To study the pattern size of nodal domains we consider the density of zeros on any straight line through the spatial domain. Using a theorem by Edelman and Kostlan and weighted ergodic theorems that ensure the convergence of the moving sums, we show that the average distance of zeros is asymptotically of order ε with a precisely given constant.

Joint work with D. Blömker and P. Düren

Christopher J. Bose (University of Victoria, BC)

Limit theorems for random intermittent maps

It is well-known that intermittent maps can be used to model unusual statistical behaviour, such as sub exponential decay of correlation, dynamical CLT, or not, stable laws, or not, depending on the strength of the intermittency. The basic question that we ask is this: if a random map is constructed by selecting from a family of intermittent maps, with a range of intermittency parameters, can we predict (or even calculate) what properties will survive into the random setting.

For example, by sampling from the family of so-called LSV maps of the interval, we have shown that the map with the fastest escape from the neutral fixed point completely determines the decay of correlation asymptotics, no matter how infrequently this map is chosen (in the i.i.d. random setting).

We continue this theme by considering CLT and stable laws for random intermittent maps, touching briefly on the more difficult problem of almost sure (or quenched) results as opposed to the more accessible annealed versions.

Based on joint work with Wael Bahsoun and Yuejiao Duan, University of Loughborough, UK.

Jiří Černý (Universität Wien)

Aging of the Metropolis dynamics of the REM

In 1992, Bouchaud proposed a toy model explaining the aging in low-temperature dynamics of spin glasses. In the last decade, many authors tried to verify that Bouchaud?s predictions are valid for the dynamics of "realistic" spin-glass models. It turned out that this verification essentially amounts to proving that certain additive functional of the Markovian dynamics converges to a stable Lévy process. This convergence was then proved for various mean-field spin-glass models, but only for the very simple Markovian dynamics, the so-called Random Hopping time dynamics, which is, however, considered to be "non-realistic".

In the talk I will present a recent result obtained jointly with T. Wassmer, proving that a similar convergence holds true also for the Metropolis dynamics of the Random Energy Model.

Robert C. Dalang (Ecole Polytechnique Fédérale de Lausanne)

Polarity of points for systems of linear stochastic pde's in critical dimensions

Given a continuous random field $X = (X_t,\, t \in \mathbb{R}^k)$ with values in $\mathbb{R}^d$, and a point $z \in \mathbb{R}^d$, it is natural to ask whether or not the hitting probability $P\{\exists t : X_t = z\}$ is positive or not. Usually, there is a critical dimension k_c such that, for $k < k_c$, the answer is "yes", for $k > k_c$, the answer is "no" (and one says that points are *polar* for X) and for $k = k_c$, the answer is more difficult to obtain. We are interested in this question when the random field is the solution of a system of d linear stochastic pde's in spatial dimension $k \geq 1$. In non-critical dimensions, the issue of polarity of points for the random field solution to these systems is well-understood. In this joint paper with C. Mueller and Y. Xiao, we extend to a wide class of anisotropic Gaussian random fields an argument developed by Talagrand (1998) for fractional Brownian motion. This allows us to establish polarity of points in critical dimensions for many systems of linear spde's, such as systems of stochastic heat and wave equations in spatial dimensions $k \geq 1$.

Sébastien Dutercq (Université d'Orléans)

Interface dynamics of a metastable mass-conserving diffusion

We will consider a diffusion process defined by the stochastic differential equation $dx_t = -\nabla V_\gamma(x_t)dt + \sqrt{(2\epsilon)}dW_t$ where V_γ is a potential with a conservation law and invariant under a group of symmetries. First we will describe the metastable states of the system, and then we will define a hierarchy on these metastable states. We will see how we can interpret the dynamics of this system in terms of the motion of its interfaces, and give sharp results on expected first-hitting times and its spectral gap.

Jan Gairing (Humboldt-Universität zu Berlin)

Exponential growth rates and ergodic theory for jump diffusions

We investigate the ergodic growth rate of (linear) jump diffusions given by a stochastic differential equation driven by a Lévy process. This rate is defined by the corresponding Lyapunov exponent. We derive a Furstenberg–Khasminskii formula expressing the top Lyapunov exponent as an ergodic average of the action of the associated generator over the sphere (the projective space). Thus ergodic properties of the projected process are considered and the uniqueness of the growth rate is derived.

Giambattista Giacomin (Université Paris Diderot)

Weak noise and non hyperbolic unstable fixed points

We consider one dimensional ordinary stochastic differential equations driven by additive Brownian motion with small variance. When the variance is zero such equations have an unstable non-hyperbolic fixed point and the drift near such a point has a power law behavior. For positive variance, the fixed point property disappears, but it is replaced by a random escape or transit time which diverges as the variance tends to zero. We show that this random time, under suitable (easily guessed) rescaling, converges to a limit random variable that essentially depends only on the power exponent associated to the fixed point. Such random variables, or laws, have therefore a universal character and they arise of course in a variety of contexts. We obtain quantitative sharp estimates, notably tail properties, on these universal laws.

This is a work in collaboration with M. Merle.

Martina Hofmanová (TU Berlin)

Stochastic mean curvature flow

Motion by mean curvature of embedded hypersurfaces in $\mathbb{R}^{N+1}$ is an important prototype of a geometric evolution law and has been intensively studied in the past decades. Mean curvature flow is characterized as a steepest descent evolution for the surface area energy and constitutes a fundamental relaxation dynamics for many problems where the interface size contributes to the systems energy. One of the main difficulties of the mean curvature flow is the appearance of topological changes and singularities in finite time. Further issues then arise in the mathematical treatment of the stochastic mean curvature flow, which was introduced as a refined model incorporating the influence of thermal noise.

We study a stochastically perturbed mean curvature flow for graphs in $\mathbb{R}^3$ over the two-dimensional unit-cube subject to periodic boundary conditions. In particular, we establish the existence of a weak martingale solution. The proof is based on energy methods and therefore presents an alternative to the stochastic viscosity solution approach. To overcome difficulties induced by the degeneracy of the mean curvature operator and the multiplicative gradient noise present in the model we employ a three step approximation scheme together with refined stochastic compactness and martingale identification methods.

Jeroen Lamb (Imperial College London)

On the problem of pitchfork bifurcation with additive noise

The problem of bifurcation in random dynamical systems has still been only very partially explored. Around the guiding examples of a one-dimensional deterministic pitchfork bifurcation with white and bounded additive noise, we propose the tools of the dichotomy spectrum, the finite-time Lyapunov spectrum, set-valued dynamics and two-point motions to provide a better understanding.

Eva Löcherbach (Université de Cergy-Pontoise)

On oscillating systems of interacting Hawkes processes

We consider multi class systems of interacting nonlinear Hawkes processes, modeling for example several large families of neurons. We prove propagation of chaos for such systems and associated central limit theorems. Moreover, we discuss situations in which the limit system exhibits oscillatory behavior. Finally, we show how these results can be related to certain PDMP's (piecewise deterministic Markov processes) and the study of their longtime behavior.

This is a joint work with Susanne Ditlevsen.

Francesca R. Nardi (Eindhoven University of Technology)

Asymptotic behavior of hitting times for Metropolis Markov chains and applications to the hard-core model on grids

We consider a stochastic model known as hard-core model where particles in a finite volume are subject to hard-core constrains and study the transition time between feasible configurations of this model. Such hard-core constraints arise in various area, such as statistical physics, combinatorics and communication networks. In the hard core model on the two-dimensional grid with periodic boundary conditions there are 2 global minima: the even configuration (respectively odd) where all particles are on the even chessboard (respectively odd). We study the tunneling time between even and odd chessboard and give estimates in probability in law and in distribution.

The work I will present is in collaboration with S. Borst, J.S.H. van Leeuwaarden and A. Zocca.

Alexandra Neamtu (Friedrich-Schiller-Universität Jena)

Dynamics of non-densely defined stochastic evolution equations

We consider a class of stochastic evolution equations

$$\begin{cases} dU(t) = AU(t)dt + F(U(t))dt + dW(t), & t \in [0, T] \\ U(0) = U_0 \in \overline{D(A)}, \end{cases}$$

with a non-densely defined linear part. In this case, the C_0-semigroup theory no longer applies. Such situations can occur due to additional restrictions which are incorporated in the domain of a linear operator. The noise term is constituted by a Banach space-valued Brownian motion. A suitable transformation allows us to reduce the stochastic equation into a random one, from which we can derive a random dynamical system and investigate the existence of random attractors. As an application, we analyze parabolic equations with nonlinear boundary conditions under stochastic influences.

Our theory is based on the integrated semigroup approach, considered in the deterministic case by P. Magal and S. Ruan.

Julian Newman (Imperial College London)

Conditions for synchronisation in random dynamical systems

We first provide necessary and sufficient conditions for a memoryless-noise random dynamical system (RDS) on a compact metric space to exhibit almost sure stable synchronisation of the trajectories of any given pair of initial conditions. We then provide necessary and sufficient conditions for a memoryless-noise RDS on a Polish space (compact or non-compact) to exhibit almost sure stable synchronisation of the trajectories of "almost any" given pair of initial conditions (where "almost" is taken with respect to an ergodic stationary distribution on the phase space); it has previously been shown by Le Jan that if the trajectories of the RDS are locally asymptotically stable, then the Markov invariant measure is supported on a finite random point-attractor (with full-measure basin of forward-attraction) – so our result provides sharp criteria for this attractor to be a singleton.

Nicolas Perkowski (Humboldt-Universität zu Berlin)

Paracontrolled KPZ equation

I will present several results that can be obtained by analyzing the KPZ equation using paracontrolled distributions. For example, we will see that its solution is given by the value function of an optimal control problem, where a Brownian motion is steered through a white noise potential. I will also compare the three available solution theories for the KPZ equation (Cole–Hopf transform, pathwise, martingale problem) and discuss how to apply them for studying scaling limits of weakly asymmetric interface growth models.

This is joint work with Massimiliano Gubinelli.

Paul Razafimandimby (Montanuniversität Leoben)

Some results on the stochastic Langrangian averaged Euler equations on non-smooth domain: Well-posedness and regularity

In this talk we will present some partial results obtained from the analysis of the stochastic Lagrangian Averaged Euler equations:

$$
\begin{aligned}
&d(\mathbf{u} - \alpha \Delta \mathbf{u}) + (\mathbf{rot}(\mathbf{u} - \alpha\Delta\mathbf{u}) \times \mathbf{u} + \nabla \tilde{\mathfrak{p}})dt = \mathbf{f} dW \quad \text{in } \mathcal{O} \times [0,T],\\
&\operatorname{div} \mathbf{u} = 0 \ \text{ in } \ \mathcal{O} \times [0,T],\\
&\mathbf{u} = 0 \text{ on } \partial\mathcal{O} \times [0,T],\\
&\mathbf{u}(0) = \mathbf{u}_0 \ \text{ in } \ \mathcal{O},
\end{aligned}
$$

where for simplicity W is a standard Brownian motion and $\mathbf{f} \in \mathbf{H}^1_0(\mathcal{O})$. We will mainly talk about the existence of $\mathbf{H}^1(\mathcal{O})$ weak martingale solution on any bounded Lipschitz domain $\mathcal{O}$. Furthermore, we show that when $\mathcal{O}$ is a convex polygon the solution $\mathbf{u}$ lives in the Sobolev space $\mathbf{W}^{2,r}(\mathcal{O})$ for some $r > 2$. We also prove that $\mathbf{rot}(\mathbf{u} - \alpha\Delta\mathbf{u})$ is continuous in $\mathrm{L}^2(\mathcal{O})$ with respect to the time variable.

Maria G. Westdickenberg (RWTH Aachen)

Local minima and energy barriers in the d-dimensional Cahn–Hilliard energy landscape

For mean values close to -1, it is easy to see that the constant state is a local energy minimizer of the Cahn–Hilliard energy. As already described in the seminal work of Cahn and Hilliard, stochastic fluctuations lead to nucleation of small, droplet-shaped regions of $+1$, which may then grow and coalesce. Moreover, whether the regions of $+1$ grow or shrink should depend on whether their mass is large enough to form a so-called critical nucleus. We describe recent (deterministic) work on the Cahn–Hilliard energy landscape in the regime of mean value close to -1 and large system size, which leads to quantitative bounds on the volume and approximate "droplet-shape" of a candidate for the critical nucleus. Our methods involve Gamma-limits, quantitative isoperimetric inequalities, and Steiner symmetrization.

Mathematical Colloquium of the Faculty of Mathematics

Thursday, 5 November 2015, 17:15 h, Room V2-210/216

Michael Loss (Georgia Institute of Technology, Atlanta, GA)

Optimal functional inequalities and flows

Functional inequalities are indispensable tools in analysis; yet, they do not form a cohesive mathematical theory. While many famous mathematicians have their names associated with one or the other inequality, their fame does not stem from this association. Nevertheless, Functional Inequalities are an attractive field as can be seen by the popularity of the book 'Inequalities' by Hardy, Littlewood and Polya. Those who work turn to this field strive to come to a deep understanding of particular examples. These are inequalities that, in their sharp form, can pose formidable problems in the Calculus of Variations, especially if one tries to answer questions such as "What is the best constant and what are the optimizers that furnish the cases of equality? Do the optimizers share the underlying symmetry of the variational problem or not, i.e., is there symmetry breaking?"

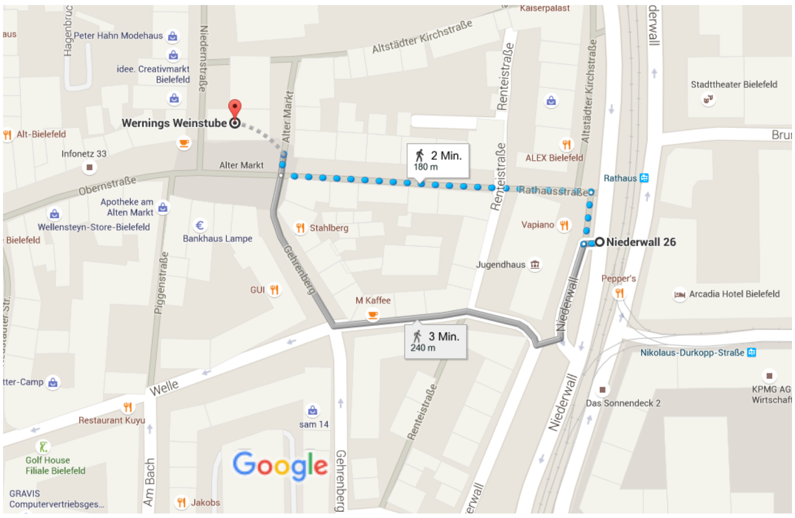
In recent years some new techniques have emerged, such as the use of flows and optimal transport theory. These not only provide elegant proofs of some known optimal inequalities; rather, they also yield new results that cannot be achieved when standard methods are used. In my talk, I will explain these techniques and illustrate them on some elementary examples. Moreover, I will discuss a recent application of the flow method that settles the phase diagram of the Caffarelli–Kohn–Nirenberg inequalities.

Preceeded by a tea at 16:30 h in V3-201.

Workshop dinner (Friday, 6 November 2015, starting at 19:00 h)

Wernings Weinstube

Alter Markt 1, 33602 Bielefeld
☎ +49 521 136 51 51
`http://www.wernings-weinstube.de`



Please note: For the dinner, prior registration is required.

Registered participants

Daniel Altemeier	Universität Bielefeld
L'ubomír Baňas	Universität Bielefeld
Manon Baudel	Université d'Orléans
Nils Berglund	Université d'Orléans
Volker Betz	TU Darmstadt
Wolf-Jürgen Beyn	Universität Bielefeld
Luigi Bianchi	Universität Augsburg
Christopher J. Bose	University of Victoria, BC
Jiří Černý	Universität Wien
Robert C. Dalang	Ecole Polytechnique Fédérale de Lausanne
Sébastien Dutercq	Université d'Orléans
Jan Gairing	Humboldt-Universität zu Berlin
Barbara Gentz	Universität Bielefeld
Giambattista Giacomin	Université Paris Diderot
Martina Hofmanová	TU Berlin *(cancelled)*
Peter Imkeller	Humboldt-Universität zu Berlin
Sabine Jansen	Ruhr-Universität Bochum
Diana Kämpfe	Universität Bielefeld
Timo Krause	Universität Bielefeld
Jeroen Lamb	Imperial College London
Eva Löcherbach	Université de Cergy-Pontoise
Francesca R. Nardi	Eindhoven University of Technology
Alexandra Neamtu	Friedrich-Schiller-Universität Jena
Julian Newman	Imperial College London
Nicolas Perkowski	Humboldt-Universität zu Berlin
Paul Razafimandimby	Montanuniversität Leoben
Deividas Sabonis	University of Copenhagen & TU München *(no-show)*
Andre Schenke	Universität Bielefeld
Maria G. Westdickenberg	RWTH Aachen
Christian Wiesel	Universität Bielefeld
Andre Wilke	Universität Bielefeld
Aelita Zilch	Universität Bielefeld

(as of 7 November 2015)