Winter term 2023/2024
Lecture: Prof. Dr. Michael Röckner
Exercises: Dr. Sebastian Grube

Exercises to Introduction to Stochastic Partial Differential Equations II

Sheet 2
Total points: 10
Submission before: Friday, 27.10.2023, 12:00 noon

Problem 1. (2 Points)

Let (X, W) be a weak solution in the sense of E.0.1. Show that $X(0)$ is independent of $\bar{W}$.

Problem 2. (4 Points)

Fill in the details of the proof of Lemma E.0.10:

Recall that

$$\mathbb{W}_0 = \{\omega \in C([0,\infty); \bar{U}) : \omega(0) = 0\}.$$

For fixed $t_0 \geqslant 0$ and $n \in \mathbb{N}$ we define for any $t_0 < t_1 < ... < t_n$ and $B_1, ..., B_n \in \mathcal{B}(\bar{U})$

$$A' := \bigcap_{k=1}^{n} \{\omega \in \mathbb{W}_0 : \pi_{t_k}(\omega) - \pi_{t_{k-1}}(\omega) \in B_k\}. \tag{$*$}$$

(a) *(1 Point)* Let A' be of the form $(*)$ (for some $n, t_0 < ... < t_n$ and $B_1, ..., B_n$). Show that $\mathbb{1}_{A'}(\bar{W})$ is P-independent of $\mathcal{F}_{t_0}$.

(b) *(1 Point)* Conclude from (a) that A' is P^Q-independent of $\mathcal{B}_{t_0}(\mathbb{W}_0)$.

(c) *(2 Points)* Prove for every $t_0 \geqslant 0$ that $\mathcal{B}(\mathbb{W}_0)$ is generated by the sets

$$\{A \cap A' : A \in \mathcal{B}_{t_0}(\mathbb{W}_0), n \in \mathbb{N}, A' \text{ as in } (*)\}.$$

Problem 3. (4 Points)

Fill in the details of the proof of Lemma E.0.11.