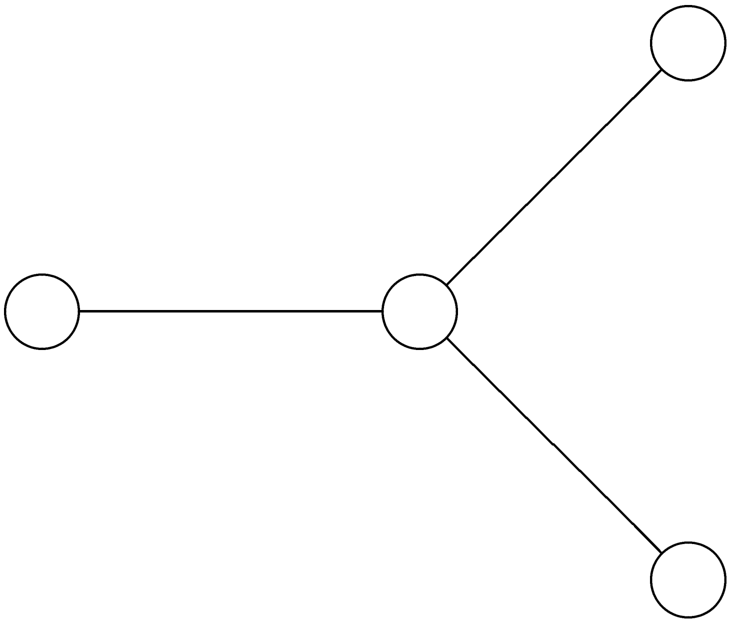
The Book of Involutions

Max-Albert Knus

Александр Сергеевич Меркурьев

Herbert Markus Rost

Jean-Pierre Tignol



The Book of Involutions

Max-Albert Knus

Alexander Merkurjev

Markus Rost

Jean-Pierre Tignol

Author address:

Dept. Mathematik, ETH-Zentrum, CH-8092 Zürich, Switzerland
E-mail address: `knus@math.ethz.ch`
URL: `http://www.math.ethz.ch/~knus/`

Dept. of Mathematics, University of California at Los Angeles, Los Angeles, California, 90095-1555, USA
E-mail address: `merkurev@math.ucla.edu`
URL: `http://www.math.ucla.edu/~merkurev/`

NWF I - Mathematik, Universität Regensburg, D-93040 Regensburg, Germany
E-mail address: `markus.rost@mathematik.uni-regensburg.de`
URL: `http://www.physik.uni-regensburg.de/~rom03516/`

Département de mathématique, Université catholique de Louvain, Chemin du Cyclotron 2, B-1348 Louvain-la-Neuve, Belgium
E-mail address: `tignol@agel.ucl.ac.be`
URL: `http://www.math.ucl.ac.be/tignol/`

Contents

Préface ix

Introduction xi

Conventions and Notations xv

Chapter I. Involutions and Hermitian Forms 1
- §1. Central Simple Algebras 3
  - 1.A. Fundamental theorems 3
  - 1.B. One-sided ideals in central simple algebras 5
  - 1.C. Severi-Brauer varieties 9
- §2. Involutions 13
  - 2.A. Involutions of the first kind 13
  - 2.B. Involutions of the second kind 20
  - 2.C. Examples 23
  - 2.D. Lie and Jordan structures 27
- §3. Existence of Involutions 31
  - 3.A. Existence of involutions of the first kind 32
  - 3.B. Existence of involutions of the second kind 36
- §4. Hermitian Forms 41
  - 4.A. Adjoint involutions 42
  - 4.B. Extension of involutions and transfer 45
- §5. Quadratic Forms 53
  - 5.A. Standard identifications 53
  - 5.B. Quadratic pairs 55
- Exercises 63
- Notes 67

Chapter II. Invariants of Involutions 71
- §6. The Index 71
  - 6.A. Isotropic ideals 72
  - 6.B. Hyperbolic involutions 74
  - 6.C. Odd-degree extensions 79
- §7. The Discriminant 80
  - 7.A. The discriminant of orthogonal involutions 80
  - 7.B. The discriminant of quadratic pairs 83
- §8. The Clifford Algebra 87
  - 8.A. The split case 87
  - 8.B. Definition of the Clifford algebra 91
  - 8.C. Lie algebra structures 95

8.D. The center of the Clifford algebra 99
8.E. The Clifford algebra of a hyperbolic quadratic pair 106
§9. The Clifford Bimodule 107
9.A. The split case 107
9.B. Definition of the Clifford bimodule 108
9.C. The fundamental relations 113
§10. The Discriminant Algebra 114
10.A. The λ-powers of a central simple algebra 115
10.B. The canonical involution 117
10.C. The canonical quadratic pair 119
10.D. Induced involutions on λ-powers 123
10.E. Definition of the discriminant algebra 127
10.F. The Brauer class of the discriminant algebra 130
§11. Trace Form Invariants 132
11.A. Involutions of the first kind 133
11.B. Involutions of the second kind 138
Exercises 145
Notes 148

Chapter III. Similitudes 153
§12. General Properties 153
12.A. The split case 153
12.B. Similitudes of algebras with involution 158
12.C. Proper similitudes 163
12.D. Functorial properties 168
§13. Quadratic Pairs 172
13.A. Relation with the Clifford structures 172
13.B. Clifford groups 177
13.C. Multipliers of similitudes 190
§14. Unitary Involutions 193
14.A. Odd degree 193
14.B. Even degree 194
14.C. Relation with the discriminant algebra 195
Exercises 199
Notes 203

Chapter IV. Algebras of Degree Four 205
§15. Exceptional Isomorphisms 205
15.A. $B_1 \equiv C_1$ 207
15.B. $A_1^2 \equiv D_2$ 210
15.C. $B_2 \equiv C_2$ 216
15.D. $A_3 \equiv D_3$ 220
§16. Biquaternion Algebras 233
16.A. Albert forms 235
16.B. Albert forms and symplectic involutions 237
16.C. Albert forms and orthogonal involutions 245
§17. Whitehead Groups 253
17.A. SK_1 of biquaternion algebras 253
17.B. Algebras with involution 266

Exercises . . . 270
Notes . . . 274

Chapter V. Algebras of Degree Three . . . 279
§18. Étale and Galois Algebras . . . 279
18.A. Étale algebras . . . 280
18.B. Galois algebras . . . 287
18.C. Cubic étale algebras . . . 296
§19. Central Simple Algebras of Degree Three . . . 302
19.A. Cyclic algebras . . . 302
19.B. Classification of involutions of the second kind . . . 304
19.C. Étale subalgebras . . . 307
Exercises . . . 319
Notes . . . 321

Chapter VI. Algebraic Groups . . . 323
§20. Hopf Algebras and Group Schemes . . . 324
20.A. Group schemes . . . 325
§21. The Lie Algebra and Smoothness . . . 334
21.A. The Lie algebra of a group scheme . . . 334
§22. Factor Groups . . . 339
22.A. Group scheme homomorphisms . . . 339
§23. Automorphism Groups of Algebras . . . 344
23.A. Involutions . . . 345
23.B. Quadratic pairs . . . 350
§24. Root Systems . . . 352
24.A. Classification of irreducible root systems . . . 354
§25. Split Semisimple Groups . . . 355
25.A. Simple split groups of type A, B, C, D, F, and G . . . 357
25.B. Automorphisms of split semisimple groups . . . 360
§26. Semisimple Groups over an Arbitrary Field . . . 360
26.A. Basic classification results . . . 364
26.B. Algebraic groups of small dimension . . . 373
§27. Tits Algebras of Semisimple Groups . . . 375
27.A. Definition of the Tits algebras . . . 376
27.B. Simply connected classical groups . . . 378
27.C. Quasisplit groups . . . 379
Exercises . . . 380
Notes . . . 381

Chapter VII. Galois Cohomology . . . 383
§28. Cohomology of Profinite Groups . . . 383
28.A. Cohomology sets . . . 383
28.B. Cohomology sequences . . . 385
28.C. Twisting . . . 387
28.D. Torsors . . . 388
§29. Galois Cohomology of Algebraic Groups . . . 391
29.A. Hilbert's Theorem 90 and Shapiro's lemma . . . 392
29.B. Classification of algebras . . . 395

29.C. Algebras with a distinguished subalgebra 398
29.D. Algebras with involution 399
29.E. Quadratic spaces 406
29.F. Quadratic pairs 408
§30. Galois Cohomology of Roots of Unity 413
30.A. Cyclic algebras 414
30.B. Twisted coefficients 416
30.C. Cohomological invariants of algebras of degree three 420
§31. Cohomological Invariants 423
31.A. Connecting homomorphisms 423
31.B. Cohomological invariants of algebraic groups 429
Exercises 442
Notes 446

Chapter VIII. Composition and Triality 451
§32. Nonassociative Algebras 451
§33. Composition Algebras 454
33.A. Multiplicative quadratic forms 454
33.B. Unital composition algebras 456
33.C. Hurwitz algebras 458
33.D. Composition algebras without identity 462
§34. Symmetric Compositions 463
34.A. Para-Hurwitz algebras 464
34.B. Petersson algebras 466
34.C. Cubic separable alternative algebras 469
34.D. Alternative algebras with unitary involutions 477
34.E. Cohomological invariants of symmetric compositions 480
§35. Clifford Algebras and Triality 481
35.A. The Clifford algebra 481
35.B. Similitudes and triality 483
35.C. The group Spin and triality 485
§36. Twisted Compositions 489
36.A. Multipliers of similitudes of twisted compositions 493
36.B. Cyclic compositions 495
36.C. Twisted Hurwitz compositions 499
36.D. Twisted compositions of type A_2' 504
36.E. The dimension 2 case 506
Exercises 507
Notes 509

Chapter IX. Cubic Jordan Algebras 513
§37. Jordan Algebras 513
37.A. Jordan algebras of quadratic forms 514
37.B. Jordan algebras of classical type 515
37.C. Freudenthal algebras 516
§38. Cubic Jordan Algebras 519
38.A. The Springer decomposition 522
§39. The Tits Construction 524
39.A. Symmetric compositions and Tits constructions 528

39.B. Automorphisms of Tits constructions 529
§40. Cohomological Invariants 533
40.A. Invariants of twisted compositions 539
§41. Exceptional Simple Lie Algebras 539
Exercises 540
Notes 542

Chapter X. Trialitarian Central Simple Algebras 547
§42. Algebras of Degree 8 547
42.A. Trialitarian triples 547
42.B. Decomposable involutions 550
§43. Trialitarian Algebras 552
43.A. A definition and some properties 552
43.B. Quaternionic trialitarian algebras 555
43.C. Trialitarian algebras of type $^2D_4$ 558
§44. Classification of Algebras and Groups of Type D_4 560
44.A. Groups of trialitarian type D_4 561
44.B. The Clifford invariant 563
§45. Lie Algebras and Triality 565
45.A. Local triality 567
45.B. Derivations of twisted compositions 569
45.C. Lie algebras and trialitarian algebras 569
Exercise 571
Notes 571

Bibliography 573

Index 585

Notation 589

Bibliography

Page references at the end of entries indicate the places in the text where the entry is quoted.

1. J. F. Adams, *Spin*(8)*, triality, F_4 and all that*, Superspace and Supergravity (S. W. Hawking and M. Roček, eds.), Cambridge University Press, Cambridge, 1981, Proceedings of the Nuffield Workshop, Cambridge, June 16 - July 12, 1980, pp. 435–445. *511.*
2. A. A. Albert, *On the Wedderburn norm condition for cyclic algebras*, Bull. Amer. Math. Soc. **37** (1931), 301–312, (also: [12, pp. 185–196]). *205, 236, 275.*
3. ———, *Normal division algebras of degree four over an algebraic field*, Trans. Amer. Math. Soc. **34** (1932), 363–372, (also: [12, pp. 279–288]). *233.*
4. ———, *On a certain algebra of quantum mechanics*, Ann. of Math. (2) **35** (1934), 65–73, (also: [13, pp. 85–93]). *516, 542.*
5. ———, *Involutorial simple algebras and real Riemann matrices*, Ann. of Math. (2) **36** (1935), 886–964, (also: [12, pp. 475–553]). *68.*
6. ———, *A structure theory for Jordan algebras*, Ann. of Math. (2) **48** (1947), 546–567, (also: [13, pp. 401–422]). *514, 516, 542.*
7. ———, *A theory of power-associative commutative algebras*, Trans. Amer. Math. Soc. **69** (1950), 503–527, (also: [13, pp. 515–539]). *543.*
8. ———, *A construction of exceptional Jordan division algebras*, Ann. of Math. (2) **67** (1958), 1–28, (also: [13, pp. 701–728]). *545.*
9. ———, *Structure of algebras*, American Mathematical Society, Providence, R.I., 1961, Revised printing of the 1939 edition. American Mathematical Society Colloquium Publications, Vol. XXIV. *67, 68, 191, 270, 321, 414, 415, 447, 555.*
10. ———, *On involutorial associative division algebras*, Scripta Math. **26** (1963), 309–316, (also: [12, pp. 679–686]). *308, 322, 527.*
11. ———, *Tensor products of quaternion algebras*, Proc. Amer. Math. Soc. **35** (1972), 65–66, (also: [12, pp. 739–740]). *251, 275.*
12. ———, *Collected mathematical papers. Part 1*, American Mathematical Society, Providence, RI, 1993, Associative algebras and Riemann matrices, Edited by R. E. Block, N. Jacobson, J. Marshall Osborn, D. J. Saltman and D. Zelinsky.
13. ———, *Collected mathematical papers. Part 2*, American Mathematical Society, Providence, RI, 1993, Nonassociative algebras and miscellany, Edited by R. E. Block, N. Jacobson, J. Marshall Osborn, D. J. Saltman and D. Zelinsky.
14. A. A. Albert and N. Jacobson, *On reduced exceptional simple Jordan algebras*, Ann. of Math. (2) **66** (1957), 400–417, (also: [13, pp. 677–694], [144, pp. 310–327]). *519, 541, 543.*
15. A. A. Albert and L. J. Paige, *On a homomorphism property of certain Jordan algebras*, Trans. Amer. Math. Soc. **93** (1959), 20–29, (also: [13, pp. 755–764]). *516.*
16. H. P. Allen, *Jordan algebras and Lie algebras of type D_4*, J. Algebra **5** (1967), 250–265. *570, 571.*
17. ———, *Hermitian forms. II*, J. Algebra **10** (1968), 503–515. *149.*
18. H. P. Allen and J. C. Ferrar, *New simple Lie algebras of type D_4*, Bull. Amer. Math. Soc. **74** (1968), 478–483. *556–558.*
19. B. N. Allison, *Lie algebras of type D_4 over number fields*, Pacific J. Math. **156** (1992), no. 2, 209–250. *555, 570–572.*
20. S. A. Amitsur, L. H. Rowen, and J.-P. Tignol, *Division algebras of degree* 4 *and* 8 *with involution*, Israel J. Math. **33** (1979), no. 2, 133–148. *27, 552.*
21. J. K. Arason, *Cohomologische Invarianten quadratischer Formen*, J. Algebra **36** (1975), no. 3, 448–491. *436, 447.*

22. J. K. Arason and R. Elman, *Nilpotence in the Witt ring*, Amer. J. Math. **113** (1991), no. 5, 861–875. *447.*
23. J. K. Arason, R. Elman, and B. Jacob, *Fields of cohomological 2-dimension three*, Math. Ann. **274** (1986), no. 4, 649–657. *276, 413.*
24. E. Artin, *Geometric algebra*, Wiley Classics Library, John Wiley & Sons Inc., New York, 1988, Reprint of the 1957 original, A Wiley-Interscience Publication. *19, 154.*
25. M. Artin, *Brauer-Severi varieties* (Notes by A. Verschoren), Brauer groups in ring theory and algebraic geometry (Wilrijk, 1981) (F. M. J. van Oystaeyen and A. H. M. J. Verschoren, eds.), Lecture Notes in Math., vol. 917, Springer, Berlin, 1982, pp. 194–210. *67, 447.*
26. M. Auslander and O. Goldman, *The Brauer group of a commutative ring*, Trans. Amer. Math. Soc. **97** (1960), 367–409. *321.*
27. R. Baer, *Linear algebra and projective geometry*, Academic Press Inc., New York, N. Y., 1952. *8.*
28. R. Baeza, *Quadratic forms over semilocal rings*, Springer-Verlag, Berlin, 1978, Lecture Notes in Mathematics, Vol. 655. *xix, 183, 299, 461, 474.*
29. A. Bak, *K-theory of forms*, Annals of Mathematics Studies, vol. 98, Princeton University Press, Princeton, N.J., 1981. *69.*
30. A. D. Barnard, *Commutative rings with operators (Galois theory and ramification)*, Proc. London Math. Soc. (3) **28** (1974), 274–290. *320.*
31. H.-J. Bartels, *Invarianten hermitescher Formen über Schiefkörpern*, Math. Ann. **215** (1975), 269–288. *448.*
32. E. Bayer-Fluckiger, *Multiplicateurs de similitudes*, C. R. Acad. Sci. Paris Sér. I Math. **319** (1994), no. 11, 1151–1153. *204.*
33. E. Bayer-Fluckiger and H. W. Lenstra, Jr., *Forms in odd degree extensions and self-dual normal bases*, Amer. J. Math. **112** (1990), no. 3, 359–373. *79, 80.*
34. E. Bayer-Fluckiger and R. Parimala, *Galois cohomology of the classical groups over fields of cohomological dimension* ≤ 2, Invent. Math. **122** (1995), no. 2, 195–229. *202, 449.*
35. ———, *Classical groups and Hasse principle*, preprint, 1998. *449.*
36. E. Bayer-Fluckiger, D. B. Shapiro, and J.-P. Tignol, *Hyperbolic involutions*, Math. Z. **214** (1993), no. 3, 461–476. *145, 149.*
37. K. I. Beĭdar, W. S. Martindale III, and A. V. Mikhalëv, *Lie isomorphisms in prime rings with involution*, J. Algebra **169** (1994), no. 1, 304–327. *68.*
38. A. Berele and D. J. Saltman, *The centers of generic division algebras with involution*, Israel J. Math. **63** (1988), no. 1, 98–118. *31.*
39. A.-M. Bergé and J. Martinet, *Formes quadratiques et extensions en caractéristique* 2, Ann. Inst. Fourier (Grenoble) **35** (1985), no. 2, 57–77. *320, 321.*
40. E. R. Berlekamp, *An analog to the discriminant over fields of characteristic two*, J. Algebra **38** (1976), no. 2, 315–317. *321.*
41. F. van der Blij, *History of the octaves*, Simon Stevin **34** (1960/1961), 106–125. *509.*
42. F. van der Blij and T. A. Springer, *The arithmetics of octaves and of the group* G_2, Nederl. Akad. Wetensch. Proc. Ser. A 62 = Indag. Math. **21** (1959), 406–418. *456, 510.*
43. ———, *Octaves and triality*, Nieuw Arch. Wisk. (3) **8** (1960), 158–169. *510, 511.*
44. D. Boos, *Ein tensorkategorieller Zugang zum Satz von Hurwitz*, Diplomarbeit, ETH Zürich, 1998.
45. A. Borel, *Linear algebraic groups*, second ed., Graduate Texts in Mathematics, vol. 126, Springer-Verlag, New York, 1991. *323, 339, 342, 347–349, 381.*
46. A. Borel and J.-P. Serre, *Théorèmes de finitude en cohomologie galoisienne*, Comment. Math. Helv. **39** (1964), 111–164. *446.*
47. A. Borel and J. Tits, *Groupes réductifs*, Inst. Hautes Études Sci. Publ. Math. (1965), no. 27, 55–150. *149, 323, 355.*
48. ———, *Compléments à l'article: "Groupes réductifs"*, Inst. Hautes Études Sci. Publ. Math. (1972), no. 41, 253–276. *438.*
49. N. Bourbaki, *Éléments de mathématique*, Masson, Paris, 1981, Algèbre. Chapitres 4 à 7. *281, 300, 338, 412, 414.*
50. ———, *Éléments de mathématique*, Masson, Paris, 1985, Algèbre commutative. Chapitres 1 à 4. *323, 339, 340.*
51. ———, *Éléments de mathématique*, Masson, Paris, 1985, Algèbre commutative. Chapitres 5 à 7. *321, 323, 331.*

52. ———, *Éléments de mathématique*, Masson, Paris, 1983, Algèbre commutative. Chapitres 8 et 9. *323.*
53. ———, *Éléments de mathématique*, Masson, Paris, 1981, Groupes et algèbres de Lie. Chapitres 4, 5 et 6. *352.*
54. ———, *Éléments de mathématique*, Hermann, Paris, 1975, Groupes et algèbres de Lie. Chapitre 7 et 8. *354.*
55. H. Braun and M. Koecher, *Jordan-Algebren*, Grundlehren der Mathematischen Wissenschaften, vol. 128, Springer-Verlag, Berlin, 1966. *542.*
56. K. S. Brown, *Cohomology of groups*, Graduate Texts in Mathematics, vol. 87, Springer-Verlag, New York, 1982. *384, 395, 417, 418, 423, 446, 555.*
57. É. Cartan, *Les groupes réels simples finis et continus*, Ann. Sci. École Norm. Sup. **31** (1914), 263–355, (also: [60, pp. 399–491]). *509.*
58. ———, *Le principe de dualité et la théorie des groupes simples et semi-simples*, Bull. Sci. Math. **49** (1925), 361–374, (also: [60, pp. 555–568]). *511.*
59. ———, *Leçons sur la théorie des spineurs*, Hermann, Paris, 1938, english transl.: The theory of spinors, Dover Publications Inc., New York, 1981. *511.*
60. ———, *Œuvres complètes. Partie I. Groupes de Lie*, Gauthier-Villars, Paris, 1952. *509, 511.*
61. A. Cayley, *Collected mathematical papers. Vol. I-XIII*, Johnson Reprint Corporation, New York and London, 1961. *509.*
62. S. U. Chase, D. K. Harrison, and A. Rosenberg, *Galois theory and Galois cohomology of commutative rings*, Mem. Amer. Math. Soc. No. **52** (1965), 15–33. *321.*
63. F. Châtelet, *Variations sur un thème de H. Poincaré*, Ann. Sci. École Norm. Sup. (3) **61** (1944), 249–300. *10, 446.*
64. ———, *Méthode galoisienne et courbes de genre un*, Ann. Univ. Lyon. Sect. A. (3) **9** (1946), 40–49. *446.*
65. V. I. Chernousov, *A remark on the* $(\bmod 5)$*-invariant of Serre for groups of type* E_8, Mat. Zametki **56** (1994), no. 1, 116–121, 157 (Russian), english transl.: Mathematical Notes (Institute of Mathematics, Belorussian Academy of Science, Minsk), Vol. **56** (1994), Nos. 1–2, 730–733. *449.*
66. C. C. Chevalley, *The algebraic theory of spinors*, Columbia University Press, New York, 1954, also: Collected Works, vol. 2, Springer-Verlag, Berlin, 1996. *481, 492, 511.*
67. C. C. Chevalley and R. D. Schafer, *The exceptional simple Lie algebras* F_4 *and* E_6, Proc. Nat. Acad. Sci. U.S.A. **36** (1950), 137–141. *359, 543, 569.*
68. I. Dejaiffe, *Somme orthogonale d'algèbres à involution et algèbre de Clifford*, Preprint, 1996. *149.*
69. P. Deligne and D. Sullivan, *Division algebras and the Hausdorff-Banach-Tarski paradox*, Enseign. Math. (2) **29** (1983), no. 1-2, 145–150. *31.*
70. M. Demazure and P. Gabriel, *Groupes algébriques. Tome I: Géométrie algébrique, généralités, groupes commutatifs*, Masson & Cie, Éditeur, Paris, 1970, Avec un appendice *Corps de classes local* par Michiel Hazewinkel. *381.*
71. F. DeMeyer and E. Ingraham, *Separable algebras over commutative rings*, Springer-Verlag, Berlin, 1971, Lecture Notes in Mathematics, Vol. 181. *321.*
72. M. Deuring, *Algebren*, Springer-Verlag, Berlin, 1968, Zweite, korrigierte Auflage. Ergebnisse der Mathematik und ihrer Grenzgebiete, Band 41. *67.*
73. H. Dherte, *Decomposition of Mal'cev-Neumann division algebras with involution*, Math. Z. **216** (1994), no. 4, 629–644. *550.*
74. L. E. Dickson, *Linear groups*, Teubner, Leipzig, 1901. *203.*
75. ———, *Linear algebras*, Trans. Amer. Math. Soc. **13** (1912), 59–73. *509.*
76. ———, *Linear algebras*, Hafner Publishing Co., New York, 1960, Reprint of Cambridge Tracts in Mathematics and Mathematical Physics, No. 16, 1914. *509.*
77. J. Dieudonné, *Les extensions quadratiques des corps non commutatifs et leurs applications*, Acta Math. **87** (1952), 175–242, (also: [82, pp. 296–363]). *274, 275.*
78. ———, *On the structure of unitary groups*, Trans. Amer. Math. Soc. **72** (1952), 367–385, (also: [82, pp. 277–295]). *266.*
79. ———, *Sur les multiplicateurs des similitudes*, Rend. Circ. Mat. Palermo (2) **3** (1954), 398–408 (1955), (also: [82, pp. 408–418]). *203.*

80. ———, *Pseudo-discriminant and Dickson invariant*, Pacific J. Math. **5** (1955), 907–910, (also: [82, pp. 419–422]). *203.*

81. ———, *Sur les groupes classiques*, Hermann, Paris, 1967, Publications de l'Institut de Mathématique de l'Université de Strasbourg, VI, Actualités Scientifiques et Industrielles, No. 1040. *178, 180.*

82. ———, *Choix d'œuvres mathématiques. Tome II*, Hermann, Paris, 1981.

83. P. K. Draxl, *Über gemeinsame separabel-quadratische Zerfällungskörper von Quaternionenalgebren*, Nachr. Akad. Wiss. Göttingen Math.-Phys. Kl. II **16** (1975), 251–259. *275.*

84. ———, *Skew fields*, London Mathematical Society Lecture Note Series, vol. 81, Cambridge University Press, Cambridge, 1983. *4, 5, 18, 25, 27, 36, 38, 39, 50, 163, 302, 303.*

85. P. K. Draxl and M. Kneser (eds.), *SK_1 von Schiefkörpern*, Lecture Notes in Mathematics, vol. 778, Berlin, Springer, 1980, Seminar held at Bielefeld and Göttingen, 1976. *253.*

86. A. Ducros, *Principe de Hasse pour les espaces principaux homogènes sous les groupes classiques sur un corps de dimension cohomologique virtuelle au plus* 1, Manuscripta Math. **89** (1996), no. 3, 335–354. *449.*

87. B. Eckmann, *Cohomology of groups and transfer*, Ann. of Math. (2) **58** (1953), 481–493. *447.*

88. A. Elduque and H. C. Myung, *Flexible composition algebras and Okubo algebras*, Comm. Algebra **19** (1991), no. 4, 1197–1227. *510.*

89. ———, *On flexible composition algebras*, Comm. Algebra **21** (1993), no. 7, 2481–2505. *463, 479, 510.*

90. A. Elduque and J. M. Pérez, *Composition algebras with associative bilinear form*, Comm. Algebra **24** (1996), no. 3, 1091–1116. *463, 467, 468, 476, 510.*

91. R. Elman and T. Y. Lam, *Pfister forms and K-theory of fields*, J. Algebra **23** (1972), 181–213. *276, 413.*

92. ———, *Classification theorems for quadratic forms over fields*, Comment. Math. Helv. **49** (1974), 373–381. *203.*

93. H. Esnault, B. Kahn, M. Levine, and E. Viehweg, *The Arason invariant and mod* 2 *algebraic cycles*, J. Amer. Math. Soc. **11** (1998), no. 1, 73–118. *449.*

94. D. K. Faddeev, *On the theory of homology in groups*, Izv. Akad. Nauk SSSR Ser. Mat. **16** (1952), 17–22. *447.*

95. J. R. Faulkner, *Finding octonion algebras in associative algebras*, Proc. Amer. Math. Soc. **104** (1988), no. 4, 1027–1030. *463, 469, 472, 510.*

96. J. R. Faulkner and J. C. Ferrar, *Exceptional Lie algebras and related algebraic and geometric structures*, Bull. London Math. Soc. **9** (1977), no. 1, 1–35. *381.*

97. J. C. Ferrar and H. P. Petersson, *Exceptional simple Jordan algebras and Galois cohomology*, Arch. Math. (Basel) **61** (1993), no. 6, 517–520. *529, 530, 545.*

98. R. W. Fitzgerald, *Witt kernels of function field extensions*, Pacific J. Math. **109** (1983), no. 1, 89–106. *276.*

99. J. Frenkel, *Cohomologie non abélienne et espaces fibrés*, Bull. Soc. Math. France **85** (1957), 135–220. *446.*

100. H. Freudenthal, *Beziehungen der E_7 und E_8 zur Oktavenebene. I*, Nederl. Akad. Wetensch. Proc. Ser. A. 57 = Indag. Math. **16** (1954), 218–230. *543.*

101. ———, *Oktaven, Ausnahmegruppen und Oktavengeometrie*, Geom. Dedicata **19** (1985), no. 1, 7–63, Reprint of Utrecht Lecture Notes, 1951. *359, 360, 509, 511, 543.*

102. A. Fröhlich and A. M. McEvett, *Forms over rings with involution*, J. Algebra **12** (1969), 79–104. *68.*

103. R. Garibaldi, *Isotropic trialitarian algebraic groups*, Preprint, 1997. *572.*

104. ———, *Clifford algebras of hyperbolic involutions*, Preprint, 1998. *150.*

105. R. Garibaldi, J.-P. Tignol, and A. R. Wadsworth, *Galois cohomology of special orthogonal groups*, Manuscripta Math. **93** (1997), no. 2, 247–266. *447, 448.*

106. P. Gille, *R-équivalence et principe de norme en cohomologie galoisienne*, C. R. Acad. Sci. Paris Sér. I Math. **316** (1993), no. 4, 315–320. *495.*

107. M. J. Greenberg, *Lectures on forms in many variables*, W. A. Benjamin, Inc., New York-Amsterdam, 1969. *521.*

108. A. Grothendieck, *A general theory of fiber spaces with structure sheaf*, Tech. report, University of Kansas, 1955. *446.*

109. ———, *Technique de descente et théorèmes d'existence en géométrie algébrique I.*, 1959/1960, Séminaire Bourbaki n°190. *321, 446.*
110. ———, *Schémas en groupes. I, II, III*, Springer-Verlag, Berlin, 1962/1964, Séminaire de Géométrie Algébrique du Bois Marie 1962/64 (SGA 3). Dirigé par M. Demazure et A. Grothendieck. Lecture Notes in Mathematics, Vol. 151, 152, 153. *381.*
111. A. J. Hahn and O. T. O'Meara, *The classical groups and K-theory*, Grundlehren der Mathematischen Wissenschaften, vol. 291, Springer-Verlag, Berlin, 1989, With a foreword by J. Dieudonné. *69.*
112. D. E. Haile, *On central simple algebras of given exponent*, J. Algebra **57** (1979), no. 2, 449–465. *150.*
113. ———, *A useful proposition for division algebras of small degree*, Proc. Amer. Math. Soc. **106** (1989), no. 2, 317–319. *303.*
114. D. E. Haile and M.-A. Knus, *On division algebras of degree* 3 *with involution*, J. Algebra **184** (1996), no. 3, 1073–1081. *308.*
115. D. E. Haile, M.-A. Knus, M. Rost, and J.-P. Tignol, *Algebras of odd degree with involution, trace forms and dihedral extensions*, Israel J. Math. **96** (1996), part B, 299–340. *322, 447.*
116. W. R. Hamilton, *Mathematical papers*, vol. 3, Cambridge University Press, 1963. *509.*
117. J. Harris, *Algebraic geometry. A first course*, Graduate Texts in Mathematics, vol. 133, Springer-Verlag, New York, 1992. *9, 11, 12.*
118. A. Held and W. Scharlau, *On the existence of involutions on simple algebras IV*, Preprint, 1995. *68.*
119. I. N. Herstein, *Topics in ring theory*, The University of Chicago Press, Chicago, Ill.-London, 1969. *68.*
120. ———, *Rings with involution*, The University of Chicago Press, Chicago, Ill.-London, 1976, Chicago Lectures in Mathematics. *68.*
121. H. Hijikata, *A remark on the groups of type G_2 and F_4*, J. Math. Soc. Japan **15** (1963), 159–164. *381.*
122. D. Hilbert, *Die Theorie der algebraischen Zahlkörper*, Jahresbericht der Deutschen Mathematikervereinigung **4** (1897), 175–546, (pp. 63–363 in: Gesammelte Abhandlungen. Bd. I., Springer-Verlag, Berlin, 1970). *393.*
123. G. Hochschild, *On the cohomology groups of an associative algebra*, Ann. of Math. (2) **46** (1945), 58–67. *321.*
124. G. Hochschild and T. Nakayama, *Cohomology in class field theory*, Ann. of Math. (2) **55** (1952), 348–366. *447.*
125. J. E. Humphreys, *Linear algebraic groups*, Springer-Verlag, New York, 1975, Graduate Texts in Mathematics, No. 21. *323, 339, 375, 381.*
126. A. Hurwitz, *Über die Komposition der quadratischen Formen von beliebigen vielen Variablen*, Nachrichten Ges. der Wiss. Göttingen, Math.-Phys. Klasse (1898), 309–316, (pp. 565–571 in: Mathematische Werke. Bd. II, Birkhäuser Verlag, Basel, 1963). *509.*
127. B. Jacob and A. Rosenberg (eds.), *K-theory and algebraic geometry: connections with quadratic forms and division algebras*, Proceedings of Symposia in Pure Mathematics, vol. 58, Part 2, Providence, RI, American Mathematical Society, 1995.
128. N. Jacobson, *Cayley numbers and normal simple Lie algebras of type G*, Duke Math. J. **5** (1939), 775–783, (also: [143, pp. 191–199]). *360, 509.*
129. ———, *A note on hermitian forms*, Bull. Amer. Math. Soc. **46** (1940), 264–268, (also: [143, pp. 208–212]). *151, 239, 518.*
130. ———, *A theorem on the structure of Jordan algebras*, Proc. Nat. Acad. Sci. U.S.A. **42** (1956), 140–147, (also: [144, pp. 290–297]). *543.*
131. ———, *Composition algebras and their automorphisms*, Rend. Circ. Mat. Palermo (2) **7** (1958), 55–80, (also: [144, pp. 341–366]). *509, 510.*
132. ———, *Some groups of transformations defined by Jordan algebras. II. Groups of type F_4*, J. reine angew. Math. **204** (1960), 74–98, (also: [144, pp. 406–430]). *510.*
133. ———, *Some groups of transformations defined by Jordan algebras. III. Groups of type E_6*, J. reine angew. Math. **207** (1961), 61–85, (also: [144, pp. 431–455]). *543.*
134. ———, *Clifford algebras for algebras with involution of type D*, J. Algebra **1** (1964), 288–300, (also: [144, pp. 516–528]). *87, 149, 150, 203.*
135. ———, *Triality and Lie algebras of type D_4*, Rend. Circ. Mat. Palermo (2) **13** (1964), 129–153, (also: [144, pp. 529–553]). *510, 570, 571.*

136. ______, *Structure and representations of Jordan algebras*, American Mathematical Society, Providence, R.I., 1968, American Mathematical Society Colloquium Publications, Vol. XXXIX. *21, 29, 275, 453, 454, 509, 514–516, 519, 531, 542, 544.*
137. ______, *Lectures on quadratic Jordan algebras*, Tata Institute of Fundamental Research, Bombay, 1969, Tata Institute of Fundamental Research Lectures on Mathematics, No. 45. *542.*
138. ______, *Exceptional Lie algebras*, Marcel Dekker Inc., New York, 1971, Lecture Notes in Pure and Applied Mathematics, 1. *381, 539, 568.*
139. ______, *Abraham Adrian Albert (1905–1972)*, Bull. Amer. Math. Soc. **80** (1974), 1075–1100, (also: [145, pp. 449–474]). *542.*
140. ______, *Lie algebras*, Dover Publications Inc., New York, 1979, Republication of the 1962 original. *29, 203, 274, 510, 549, 566, 568.*
141. ______, *Structure theory of Jordan algebras*, University of Arkansas Lecture Notes in Mathematics, vol. 5, University of Arkansas, Fayetteville, Ark., 1981. *517, 520, 542.*
142. ______, *Some applications of Jordan norms to involutorial simple associative algebras*, Adv. in Math. **48** (1983), no. 2, 149–165, (also: [145, pp. 251–267]). *275.*
143. ______, *Collected mathematical papers. Vol. 1*, Contemporary Mathematicians, Birkhäuser Boston Inc., Boston, MA, 1989, (1934–1946).
144. ______, *Collected mathematical papers. Vol. 2*, Contemporary Mathematicians, Birkhäuser Boston Inc., Boston, MA, 1989, (1947–1965).
145. ______, *Collected mathematical papers. Vol. 3*, Contemporary Mathematicians, Birkhäuser Boston Inc., Boston, MA, 1989, (1965–1988). *275.*
146. ______, *Finite-dimensional division algebras over fields*, Springer-Verlag, Berlin, 1996. *31, 67, 191, 303, 313, 320.*
147. P. Jordan, J. von Neumann, and E. Wigner, *On an algebraic generalization of the quantum mechanical formalism*, Ann. of Math. (2) **35** (1934), 29–64. *542.*
148. B. Kahn, M. Rost, and R. Sujatha, *Unramified cohomology of quadrics I*, 1997, to appear in Amer. J. Math. *276.*
149. I. Kaplansky, *Infinite-dimensional quadratic forms admitting composition*, Proc. Amer. Math. Soc. **4** (1953), 956–960. *456, 462.*
150. ______, *Linear algebra and geometry. A second course*, Allyn and Bacon Inc., Boston, Mass., 1969. *30, 52.*
151. N. Karpenko and A. Quéguiner, *A criterion of decomposability for degree* 4 *algebras with unitary involution*, Preprint, 1997. *270.*
152. I. Kersten, *On involutions of the first kind*, Diskrete Strukturen in der Mathematik, vol. 343, 1996, Preprintreihe des SFB, pp. 96–107. *149.*
153. I. Kersten and U. Rehmann, *Generic splitting of reductive groups*, Tôhoku Math. J. (2) **46** (1994), no. 1, 35–70. *149.*
154. J. Kirmse, *Über die Darstellbarkeit natürlicher ganzer Zahlen als Summen von acht Quadraten und über ein mit diesem Problem zusammenhängendes nichtkommutatives und nichtassoziatives Zahlensystem*, Berichte über die Verh. der Sächs. Akad. der Wiss. Leipzig, math.-phys. Klasse **76** (1924), 63–82. *509.*
155. M. Kneser, *Lectures on Galois cohomology of classical groups*, Tata Institute of Fundamental Research, Bombay, 1969, With an appendix by T. A. Springer, Notes by P. Jothilingam, Tata Institute of Fundamental Research Lectures on Mathematics, No. 47. *50, 68, 204, 381, 449.*
156. M.-A. Knus, *Pfaffians and quadratic forms*, Adv. in Math. **71** (1988), no. 1, 1–20. *274, 275.*
157. ______, *Quadratic and Hermitian forms over rings*, Grundlehren der Mathematischen Wissenschaften, vol. 294, Springer-Verlag, Berlin, 1991, With a foreword by I. Bertuccioni. *xix, 45, 68, 69, 87–89, 96, 177, 274, 275.*
158. ______, *Sur la forme d'Albert et le produit tensoriel de deux algèbres de quaternions*, Bull. Soc. Math. Belg. Sér. B **45** (1993), no. 3, 333–337. *275.*
159. M.-A. Knus, T. Y. Lam, D. B. Shapiro, and J.-P. Tignol, *Discriminants of involutions on biquaternion algebras*, In Jacob and Rosenberg [127], pp. 279–303. *275.*
160. M.-A. Knus and M. Ojanguren, *Théorie de la descente et algèbres d'Azumaya*, Springer-Verlag, Berlin, 1974, Lecture Notes in Mathematics, Vol. 389. *32, 344.*
161. M.-A. Knus, R. Parimala, and R. Sridharan, *A classification of rank* 6 *quadratic spaces via pfaffians*, J. reine angew. Math. **398** (1989), 187–218. *275.*

162. ———, *Involutions on rank* 16 *central simple algebras*, J. Indian Math. Soc. (N.S.) **57** (1991), no. 1-4, 143–151. *27, 274.*

163. ———, *On the discriminant of an involution*, Bull. Soc. Math. Belg. Sér. A **43** (1991), no. 1-2, 89–98, Contact Franco-Belge en Algèbre (Antwerp, 1990). *149, 275.*

164. ———, *Pfaffians, central simple algebras and similitudes*, Math. Z. **206** (1991), no. 4, 589–604. *149, 271, 274, 275.*

165. ———, *On compositions and triality*, J. reine angew. Math. **457** (1994), 45–70. *511, 560.*

166. M.-A. Knus, R. Parimala, and V. Srinivas, *Azumaya algebras with involutions*, J. Algebra **130** (1990), no. 1, 65–82. *64.*

167. N. H. Kuiper, *On linear families of involutions*, Amer. J. Math. **72** (1950), 425–441. *511.*

168. V. V. Kursov, *Commutators of the multiplicative group of a finite-dimensional central division algebra*, Dokl. Akad. Nauk BSSR **26** (1982), no. 2, 101–103, 187. *277.*

169. T. Y. Lam, *The algebraic theory of quadratic forms*, W. A. Benjamin, Inc., Reading, Mass., 1973, Mathematics Lecture Note Series. *88, 141, 142, 235, 243, 439, 518, 536.*

170. T. Y. Lam, D. B. Leep, and J.-P. Tignol, *Biquaternion algebras and quartic extensions*, Inst. Hautes Études Sci. Publ. Math. (1993), no. 77, 63–102. *68, 276, 413.*

171. S. Lang and J. Tate, *Principal homogeneous spaces over abelian varieties*, Amer. J. Math. **80** (1958), 659–684. *446.*

172. D. W. Lewis, *A note on trace forms and involutions of the first kind*, Exposition. Math. **15** (1997), no. 3, 265–272. *151.*

173. D. W. Lewis and J. F. Morales, *The Hasse invariant of the trace form of a central simple algebra*, Théorie des nombres, Années 1992/93–1993/94, Publ. Math. Fac. Sci. Besançon, Univ. Franche-Comté, Besançon, 1994, pp. 1–6. *142.*

174. D. W. Lewis and J.-P. Tignol, *On the signature of an involution*, Arch. Math. (Basel) **60** (1993), no. 2, 128–135. *151.*

175. O. Loos, *Tensor products and discriminants of unital quadratic forms over commutative rings*, Monatsh. Math. **122** (1996), no. 1, 45–98. *499, 500.*

176. S. MacLane, *Categories for the working mathematician*, Springer-Verlag, New York, 1971, Graduate Texts in Mathematics, Vol. 5. *170, 361.*

177. P. Mammone and D. B. Shapiro, *The Albert quadratic form for an algebra of degree four*, Proc. Amer. Math. Soc. **105** (1989), no. 3, 525–530. *275.*

178. W. S. Martindale III, *Lie and Jordan mappings in associative rings*, Ring theory (Proc. Conf., Ohio Univ., Athens, Ohio, 1976), Dekker, New York, 1977, pp. 71–84. Lecture Notes in Pure and Appl. Math., Vol. 25. *68.*

179. H. Matsumura, *Commutative algebra*, second ed., Mathematics Lecture Note Series, vol. 56, Benjamin/Cummings Publishing Co., Inc., Reading, Mass., 1980. *323, 337.*

180. S. Maurer, *Vektorproduktalgebren*, Diplomarbeit, Universität Regensburg, 1998.

181. K. McCrimmon, *Jordan book*, `http://www.math.virginia.edu/Faculty/KMcCrimmon/`. *542.*

182. ———, *Norms and noncommutative Jordan algebras*, Pacific J. Math. **15** (1965), 925–956. *453.*

183. ———, *A general theory of Jordan rings*, Proc. Nat. Acad. Sci. U.S.A. **56** (1966), 1072–1079. *542.*

184. ———, *Generically algebraic algebras*, Trans. Amer. Math. Soc. **127** (1967), 527–551. *452, 509.*

185. ———, *The Freudenthal-Springer-Tits constructions of exceptional Jordan algebras*, Trans. Amer. Math. Soc. **139** (1969), 495–510. *519, 520, 525, 543.*

186. ———, *The Freudenthal-Springer-Tits constructions revisited*, Trans. Amer. Math. Soc. **148** (1970), 293–314. *521, 526, 544.*

187. ———, *Jordan algebras and their applications*, Bull. Amer. Math. Soc. **84** (1978), no. 4, 612–627. *542.*

188. ———, *The Russian revolution in Jordan algebras*, Algebras Groups Geom. **1** (1984), 1–61. *542.*

189. A. S. Merkurjev, *On the norm residue symbol of degree* 2, Dokl. Akad. Nauk SSSR **261** (1981), no. 3, 542–547, english transl.: Soviet Math. Dokl. **24** (1981), 546–551. *27, 45, 436.*

190. ———, *K-theory of simple algebras*, In Jacob and Rosenberg [127], pp. 65–83. *276, 277.*

191. ———, *The norm principle for algebraic groups*, Algebra i Analiz **7** (1995), no. 2, 77–105, english transl.: St. Petersburg Math. J. **7** (1996), no. 2, 243–264. *202.*

192. ———, *Maximal indexes of Tits algebras*, Doc. Math. **1** (1996), No. 12, 229–243 (electronic). *382.*

193. A. S. Merkurjev, I. A. Panin, and A. R. Wadsworth, *Index reduction formulas for twisted flag varieties. II*, 1997, to appear. *149, 382.*

194. ———, *Index reduction formulas for twisted flag varieties. I*, K-Theory **10** (1996), no. 6, 517–596. *149, 382.*

195. A. S. Merkurjev and A. A. Suslin, *K-cohomology of Severi-Brauer varieties and the norm residue homomorphism*, Izv. Akad. Nauk SSSR Ser. Mat. **46** (1982), no. 5, 1011–1046, 1135–1136, english transl.: Math. USSR Izv. **21** (1983), 307–340. *436.*

196. A. S. Merkurjev and J.-P. Tignol, *The multipliers of similitudes and the Brauer group of homogeneous varieties*, J. reine angew. Math. **461** (1995), 13–47. *132, 141, 147, 149, 150, 203, 204, 274.*

197. A. P. Monastyrnyĭ and V. I. Yanchevskiĭ, *Whitehead groups of spinor groups*, Izv. Akad. Nauk SSSR Ser. Mat. **54** (1990), no. 1, 60–96, 221, english transl.: Math. USSR Izv. **36** (1991), 61–100. *277.*

198. P. J. Morandi and B. A. Sethuraman, *Kummer subfields of tame division algebras*, J. Algebra **172** (1995), no. 2, 554–583. *273.*

199. S. Okubo, *Pseudo-quaternion and pseudo-octonion algebras*, Hadronic J. **1** (1978), no. 4, 1250–1278. *463, 472, 510.*

200. ———, *Introduction to octonion and other non-associative algebras in physics*, Montroll Memorial Lecture Series in Mathematical Physics, vol. 2, Cambridge University Press, Cambridge, 1995. *510.*

201. S. Okubo and H. C. Myung, *Some new classes of division algebras*, J. Algebra **67** (1980), no. 2, 479–490. *511.*

202. S. Okubo and J. M. Osborn, *Algebras with nondegenerate associative symmetric bilinear forms permitting composition*, Comm. Algebra **9** (1981), no. 12, 1233–1261. *464, 465, 510.*

203. L. J. Paige, *Jordan algebras*, Studies in Modern Algebra (A. A. Albert, ed.), Mathematical Association of America, Prentice-Hall, N. J., 1963, pp. 144–186. *542.*

204. G. Papy, *Sur l'arithmétique dans les algèbres de Grassman*, Acad. Roy. Belgique. Cl. Sci. Mém. Coll. 8° **26** (1952), no. 8, 108. *151.*

205. R. Parimala, R. Sridharan, and V. Suresh, *A question on the discriminants of involutions of central division algebras*, Math. Ann. **297** (1993), no. 4, 575–580. *149, 275, 552.*

206. R. Parimala, R. Sridharan, and M. L. Thakur, *A classification theorem for Albert algebras*, Trans. Amer. Math. Soc. **350** (1998), no. 3, 1277–1284. *202, 535, 538.*

207. H. P. Petersson, *Eine Identität fünften Grades, der gewisse Isotope von Kompositions-Algebren genügen*, Math. Z. **109** (1969), 217–238. *463, 466, 467, 510.*

208. ———, *Quasi composition algebras*, Abh. Math. Sem. Univ. Hamburg **35** (1971), 215–222. *463, 507, 510.*

209. H. P. Petersson and M. L. Racine, *Cubic subfields of exceptional simple Jordan algebras*, Proc. Amer. Math. Soc. **91** (1984), no. 1, 31–36. *543.*

210. ———, *Springer forms and the first Tits construction of exceptional Jordan division algebras*, Manuscripta Math. **45** (1984), no. 3, 249–272. *523, 536, 544.*

211. ———, *Classification of algebras arising from the Tits process*, J. Algebra **98** (1986), no. 1, 244–279. *532, 533, 543–545.*

212. ———, *Jordan algebras of degree* 3 *and the Tits process*, J. Algebra **98** (1986), no. 1, 211–243. *520, 525, 543, 544.*

213. ———, *Albert algebras*, Jordan algebras (Oberwolfach, 1992) (W. Kaup and K. McCrimmon, eds.), de Gruyter, Berlin, 1994, pp. 197–207. *528, 537, 543.*

214. ———, *On the invariants mod* 2 *of Albert algebras*, J. Algebra **174** (1995), no. 3, 1049–1072. *499.*

215. ———, *An elementary approach to the Serre-Rost invariant of Albert algebras*, Indag. Mathem., N.S. **7** (1996), no. 3, 343–365. *537, 545.*

216. ———, *The Serre-Rost invariant of Albert algebras in characteristic* 3, Indag. Mathem., N.S. **8** (1997), no. 4, 543–548. *545.*

217. A. Pfister, *Quadratische Formen in beliebigen Körpern*, Invent. Math. **1** (1966), 116–132. *275.*

218. ———, *Quadratic forms with applications to algebraic geometry and topology*, London Mathematical Society Lecture Note Series, vol. 217, Cambridge University Press, Cambridge, 1995. *243, 556.*
219. R. S. Pierce, *Associative algebras*, Graduate Texts in Mathematics, vol. 88, Springer-Verlag, New York, 1982, Studies in the History of Modern Science, 9. *5, 253, 267, 397, 414, 415.*
220. V. P. Platonov, *The Dieudonné conjecture, and the nonsurjectivity of coverings of algebraic groups at k-points*, Dokl. Akad. Nauk SSSR **216** (1974), 986–989, english transl.: Soviet Math. Dokl. **15** (1974), 927–931. *68.*
221. ———, *The Tannaka-Artin problem, and reduced K-theory*, Izv. Akad. Nauk SSSR Ser. Mat. **40** (1976), no. 2, 227–261, 469, english transl.: Math. USSR Izv. **10** (1976), no. 2, 211–243. *277.*
222. V. P. Platonov and A. Rapinchuk, *Algebraic groups and number theory*, Pure and Applied Mathematics, vol. 139, Academic Press Inc., Boston, MA, 1994, Translated from the 1991 Russian original by Rachel Rowen. *381, 449.*
223. V. P. Platonov and V. I. Yanchevskiĭ, *Finite-dimensional division algebras*, Algebra IX, Finite Groups of Lie Type, Finite-Dimensional Division Algebras (Berlin) (A. I. Kostrikin and I. R. Shafarevich, eds.), Encyclopaedia of Mathematical Sciences, vol. 77, Springer-Verlag, 1995, pp. 121–233. *266.*
224. A. Quéguiner, *Invariants d'algèbres à involution*, Thèse, Besançon, 1996. *151.*
225. ———, *Signature des involutions de deuxième espèce*, Arch. Math. (Basel) **65** (1995), no. 5, 408–412. *151.*
226. ———, *Cohomological invariants of algebras with involution*, J. Algebra **194** (1997), no. 1, 299–330. *151, 448.*
227. ———, *On the determinant class of an algebra with unitary involution*, Prépublications Mathématiques de l'Université Paris 13, 1998. *151, 446, 448.*
228. M. L. Racine, *A note on quadratic Jordan algebras of degree* 3, Trans. Amer. Math. Soc. **164** (1972), 93–103. *234, 521.*
229. I. Reiner, *Maximal orders*, Academic Press, London-New York, 1975, London Mathematical Society Monographs, No. 5. *5, 22, 67.*
230. P. Revoy, *Remarques sur la forme trace*, Linear and Multilinear Algebra **10** (1981), no. 3, 223–233. *321.*
231. C. Riehm, *The corestriction of algebraic structures*, Invent. Math. **11** (1970), 73–98. *68, 420.*
232. C. Rosati, *Sulle matrici di Riemann*, Rend. Circ. Mat. Palermo **53** (1929), 79–134. *68.*
233. M. P. Rosen, *Isomorphisms of a certain class of prime Lie rings*, J. Algebra **89** (1984), no. 2, 291–317. *68.*
234. M. Rost, *A* (mod 3) *invariant for exceptional Jordan algebras*, C. R. Acad. Sci. Paris Sér. I Math. **313** (1991), no. 12, 823–827. *448, 537, 545.*
235. ———, *On the dimension of a composition algebra*, Doc. Math. **1** (1996), No. 10, 209–214 (electronic).
236. L. H. Rowen, *Central simple algebras*, Israel J. Math. **29** (1978), no. 2-3, 285–301. *68.*
237. ———, *Ring theory. Vol. II*, Pure and Applied Mathematics, vol. 128, Academic Press Inc., Boston, MA, 1988. *32, 36, 39.*
238. L. H. Rowen and D. J. Saltman, *The discriminant of an involution*, Preprint, 1990. *63.*
239. C. H. Sah, *Symmetric bilinear forms and quadratic forms*, J. Algebra **20** (1972), 144–160. *275.*
240. D. J. Saltman, *The Brauer group is torsion*, Proc. Amer. Math. Soc. **81** (1981), no. 3, 385–387. *150.*
241. ———, *A note on generic division algebras*, Abelian groups and noncommutative rings (L. Fuchs, K. R. Goodearl, J. T. Stafford, and C. Vinsonhaler, eds.), Contemp. Math., vol. 130, Amer. Math. Soc., Providence, RI, 1992, A collection of papers in memory of Robert B. Warfield, Jr, pp. 385–394. *65, 550.*
242. R. D. Schafer, *The exceptional simple Jordan algebras*, Amer. J. Math. **70** (1948), 82–94. *517, 543.*
243. ———, *Forms permitting composition*, Advances in Math. **4** (1970), 127–148 (1970). *474.*
244. ———, *An introduction to nonassociative algebras*, Dover Publications Inc., New York, 1995, Corrected reprint of the 1966 original. *453, 454, 539, 569.*
245. W. Scharlau, *Induction theorems and the structure of the Witt group*, Invent. Math. **11** (1970), 37–44. *79.*

246. ______, *Zur Existenz von Involutionen auf einfachen Algebren*, Math. Z. **145** (1975), no. 1, 29–32. *68.*

247. ______, *Quadratic and Hermitian forms*, Grundlehren der mathematischen Wissenschaften, vol. 270, Springer-Verlag, Berlin, 1985. *4, 5, 19, 25, 36, 38, 50, 68, 69, 77, 79, 88, 136, 142, 144, 162, 163, 177, 178, 187, 201, 234, 243, 261, 276, 299, 305, 317, 318, 481, 483, 521, 524, 550.*

248. C. Scheiderer, *Real and étale cohomology*, Lecture Notes in Mathematics, vol. 1588, Springer-Verlag, Berlin, 1994. *132.*

249. ______, *Hasse principles and approximation theorems for homogeneous spaces over fields of virtual cohomological dimension one*, Invent. Math. **125** (1996), no. 2, 307–365. *449.*

250. E. A. M. Seip-Hornix, *Clifford algebras of quadratic quaternion forms. I, II*, Nederl. Akad. Wetensch. Proc. Ser. A 68 = Indag. Math. **27** (1965), 326–363. *69, 149.*

251. G. B. Seligman, *Modular Lie algebras*, Springer-Verlag New York, Inc., New York, 1967, Ergebnisse der Mathematik und ihrer Grenzgebiete, Band 40. *381.*

252. ______, *Rational methods in Lie algebras*, Marcel Dekker Inc., New York, 1976, Lecture Notes in Pure and Applied Mathematics, Vol. 17. *275.*

253. J.-P. Serre, *Espaces fibrés algébriques*, 1958, Séminaire Chevalley, No. 2, Exposé 1. *321.*

254. ______, *Corps locaux*, Hermann, Paris, 1968, Deuxième édition, Publications de l'Université de Nancago, No. VIII, english transl.: Local fields, Graduate Texts in Mathematics 67, Springer-Verlag, New York, 1979. *446.*

255. ______, *L'invariant de Witt de la forme* $\mathrm{Tr}(x^2)$, Comment. Math. Helv. **59** (1984), no. 4, 651–676, (also: [256, pp. 675–700]). *321.*

256. ______, *Œuvres. Vol. III*, Springer-Verlag, Berlin, 1986, 1972–1984.

257. ______, *Cohomologie galoisienne, cinquième édition, révisée et complétée*, fifth ed., Lecture Notes in Mathematics, vol. 5, Springer-Verlag, Berlin, 1994, english transl.: Galois cohomology, Springer-Verlag, Berlin, 1997. *142, 394, 446, 447, 449, 545.*

258. ______, *Cohomologie galoisienne: progrès et problèmes*, Astérisque (1995), no. 227, Exp. No. 783, 4, 229–257, Séminaire Bourbaki, Vol. 1993/94. *446, 449, 462, 517, 543, 545.*

259. D. B. Shapiro, *Symmetric and skew-symmetric linear transformations*, in preparation. *69.*

260. D. Soda, *Some groups of type D_4 defined by Jordan algebras*, J. reine angew. Math. **223** (1966), 150–163. *533.*

261. A. Speiser, *Zahlentheoretische Sätze aus der Gruppentheorie*, Math. Z. **5** (1919), 1–6. *393.*

262. T. A. Springer, *On the equivalence of quadratic forms*, Nederl. Akad. Wetensch. Proc. Ser. A 62 = Indag. Math. **21** (1959), 241–253. *446.*

263. ______, *On a class of Jordan algebras*, Nederl. Akad. Wetensch. Proc. Ser. A 62 = Indag. Math. **21** (1959), 254–264. *543.*

264. ______, *The projective octave plane. I, II*, Nederl. Akad. Wetensch. Proc. Ser. A 63 = Indag. Math. **22** (1960), 74–101. *543.*

265. ______, *The classification of reduced exceptional simple Jordan algebras*, Nederl. Akad. Wetensch. Proc. Ser. A 63 = Indag. Math. **22** (1960), 414–422. *517, 518, 543.*

266. ______, *Characterization of a class of cubic forms*, Nederl. Akad. Wetensch. Proc. Ser. A 65 = Indag. Math. **24** (1962), 259–265. *543.*

267. ______, *On the geometric algebra of the octave planes*, Nederl. Akad. Wetensch. Proc. Ser. A 65 = Indag. Math. **24** (1962), 451–468. *543.*

268. ______, *Oktaven, Jordan-Algebren und Ausnahmegruppen*, Mathematisches Institut der Universität Göttingen, 1963, english transl. in preparation. *359, 360, 495, 500, 509–511, 523, 540, 543, 544.*

269. ______, *Jordan algebras and algebraic groups*, Springer-Verlag, New York, 1973, Ergebnisse der Mathematik und ihrer Grenzgebiete, Band 75, new edition to appear. *359, 542.*

270. ______, *Linear algebraic groups*, Progress in Mathematics, vol. 9, Birkhäuser Boston, Mass., 1981, new edition to appear. *323, 381.*

271. ______, *Linear algebraic groups*, Algebraic Geometry IV, Linear Algebraic Groups, Invariant Theory (Berlin) (A. N. Parshin and I. R. Shafarevich, eds.), Encyclopaedia of Mathematical Sciences, vol. 55, Springer-Verlag, 1994, pp. 1–121. *323, 381.*

272. C. Stampfli-Rollier, 4-*dimensionale Quasikompositionsalgebren*, Arch. Math. (Basel) **40** (1983), no. 6, 516–525. *510.*

273. E. Steinitz, *Algebraische theorie der Körper*, J. reine angew. Math. **137** (1910), 167–309, (New editions: de Gruyter, Berlin, 1930, Chelsea, New York, 1950, H. Hasse, R. Baer, eds.). *321.*

274. E. Study, *Grundlagen und Ziele der analytischen Kinematik*, Sitz. Ber. Berliner Math. Gesellschaft **12** (1913), 36–60. *511.*

275. A. A. Suslin (ed.), *Algebraic K-theory*, Advances in Soviet Mathematics, vol. 4, Providence, RI, American Mathematical Society, 1991, Papers from the seminar held at Leningrad State University, Leningrad, Translation from the Russian edited by A. B. Sossinsky.

276. ———, *K-theory and K-cohomology of certain group varieties*, In *Algebraic K-theory* [275], pp. 53–74. *151, 449.*

277. ———, *SK_1 of division algebras and Galois cohomology*, In *Algebraic K-theory* [275], pp. 75–99. *277.*

278. T. Tamagawa, *On Clifford algebra*, Unpublished notes, $\sim$1971. *150.*

279. ———, *On quadratic forms and pfaffians*, J. Fac. Sci. Univ. Tokyo Sect. IA Math. **24** (1977), no. 1, 213–219. *275.*

280. D. Tao, *A variety associated to an algebra with involution*, J. Algebra **168** (1994), no. 2, 479–520. *149.*

281. ———, *The generalized even Clifford algebra*, J. Algebra **172** (1995), no. 1, 184–204. *93, 150, 550.*

282. J. Tate, *WC-groups over $\mathfrak{p}$-adic fields*, Secrétariat mathématique, Paris, 1958, Séminaire Bourbaki; 10e année: 1957/1958. Textes des conférences; Exposés 152 à 168; 2e éd. corrigée, Exposé 156, 13 pp. *446.*

283. R. C. Thompson, *Commutators in the special and general linear groups*, Trans. Amer. Math. Soc. **101** (1961), 16–33. *277.*

284. J.-P. Tignol, *On the corestriction of central simple algebras*, Math. Z. **194** (1987), no. 2, 267–274. *39.*

285. ———, *La norme des espaces quadratiques et la forme trace des algèbres simples centrales*, Théorie des nombres, Années 1992/93–1993/94, Publ. Math. Fac. Sci. Besançon, Univ. Franche-Comté, Besançon, 1994, 19 pp. *142.*

286. J. Tits, *Sur la trialité et les algèbres d'octaves*, Acad. Roy. Belg. Bull. Cl. Sci. (5) **44** (1958), 332–350. *511.*

287. ———, *Sur la trialité et certains groupes qui s'en déduisent*, Inst. Hautes Études Sci. Publ. Math. (1959), no. 2, 13–60. *511.*

288. ———, *A theorem on generic norms of strictly power associative algebras*, Proc. Amer. Math. Soc. **15** (1964), 35–36. *452.*

289. ———, *Algèbres alternatives, algèbres de Jordan et algèbres de Lie exceptionnelles. I. Construction*, Nederl. Akad. Wetensch. Proc. Ser. A 69 = Indag. Math. **28** (1966), 223–237. *539.*

290. ———, *Classification of algebraic semisimple groups*, Algebraic Groups and Discontinuous Subgroups (Proc. Sympos. Pure Math., Boulder, Colo., 1965) (A. Borel and G. D. Mostow, eds.), vol. 9, Amer. Math. Soc., Providence, R.I., 1966, 1966, pp. 33–62. *149, 323, 355.*

291. ———, *Formes quadratiques, groupes orthogonaux et algèbres de Clifford*, Invent. Math. **5** (1968), 19–41. *xii, 68, 69, 87, 146, 149, 150, 203, 323, 381.*

292. ———, *Représentations linéaires irréductibles d'un groupe réductif sur un corps quelconque*, J. reine angew. Math. **247** (1971), 196–220. *150, 151, 375, 378.*

293. ———, *Sur les produits tensoriels de deux algèbres de quaternions*, Bull. Soc. Math. Belg. Sér. B **45** (1993), no. 3, 329–331. *275.*

294. F. Vaney, *Le parallélisme absolu dans les espaces elliptiques réels à* 3 *et à* 7 *dimensions et le principe de trialité*, Thèse, Paris, 1929. *511.*

295. F. D. Veldkamp, *Freudenthal and the octonions*, Nieuw Arch. Wisk. (4) **9** (1991), no. 2, 145–162. *509.*

296. O. Villa, *On division algebras of degree* 3 *over a field of characteristic* 3, Master's thesis, ETH, Zurich, March 1997. *321.*

297. A. R. Wadsworth, Oral communication, July 1992. *550.*

298. ———, *Discriminants in characteristic two*, Linear and Multilinear Algebra **17** (1985), no. 3-4, 235–263. *321.*

299. ———, 16-*dimensional algebras with involution*, `http://math.ucsd.edu/~wadswrth/`, Unpublished notes, 1990. *246, 271.*

300. B. L. van der Waerden, *Algebra I*, Grundlehren der Mathematischen Wissenschaften, vol. 33, Springer-Verlag, Berlin, 1930. *321.*
301. ———, *Gruppen von linearen Transformationen*, Springer-Verlag, Berlin, 1935, Ergebnisse der Mathematik und ihrer Grenzgebiete, Band 4. *274, 275.*
302. ———, *A history of algebra, From al-Khwārizmī to Emmy Noether*, Springer-Verlag, Berlin, 1985. *509.*
303. R. Walde, *Jordan algebras and exceptional Lie algebras*, Preprint, 1967. *544.*
304. S. Wang, *On the commutator subgroup of a simple algebra*, Amer. J. Math. **72** (1950), 323–334. *253, 267.*
305. W. C. Waterhouse, *Introduction to affine group schemes*, Graduate Texts in Mathematics, vol. 66, Springer-Verlag, New York, 1979. *281, 323–325, 330, 335, 338–340, 343, 357.*
306. ———, *Discriminants of étale algebras and related structures*, J. reine angew. Math. **379** (1987), 209–220. *321.*
307. J. H. M. Wedderburn, *On division algebras*, Trans. Amer. Math. Soc. **22** (1921), 129–135. *321.*
308. A. Weil, *On algebraic groups and homogeneous spaces*, Amer. J. Math. **77** (1955), 493–512, (also: [311, pp. 235–254]). *446.*
309. ———, *The field of definition of a variety*, Amer. J. Math. **78** (1956), 509–524, (also: [311, pp. 291–306]). *446.*
310. ———, *Algebras with involutions and the classical groups*, J. Indian Math. Soc. (N.S.) **24** (1960), 589–623 (1961), (also: [311, pp. 413–447]). *xii, 151, 323, 381, 446.*
311. ———, *Scientific works. Collected papers. Vol. II (1951–1964)*, Springer-Verlag, New York, 1979. *381, 446.*
312. E. A. Weiss, *Oktaven, Engelscher Komplex, Trialitätsprinzip*, Math. Z. **44** (1938), 580–611. *511.*
313. ———, *Punktreihengeometrie*, Teubner, Leipzig, Berlin, 1939. *511.*
314. H. Weyl, *On generalized Riemann matrices*, Ann. of Math. (2) **35** (1934), 714–729, (also: [315, Vol. III, pp. 400–415]). *67, 68.*
315. ———, *Gesammelte Abhandlungen. Bände I, II, III, IV*, Springer-Verlag, Berlin, 1968, Herausgegeben von K. Chandrasekharan.
316. M. J. Wonenburger, *The Clifford algebra and the group of similitudes*, Canad. J. Math. **14** (1962), 45–59. *175, 200, 203.*
317. ———, *Triality principle for semisimilarities*, J. Algebra **1** (1964), 335–341. *510.*
318. V. I. Yanchevskiĭ, *Simple algebras with involutions, and unitary groups*, Mat. Sb. (N.S.) **93 (135)** (1974), 368–380, 487, english transl.: Math. USSR Sb. **22** (1974), 372–385. *277.*
319. ———, *Symmetric and skew-symmetric elements of involutions, associated groups and the problem of decomposability of involutions*, In Jacob and Rosenberg [127], pp. 431–444. *148, 266, 268, 277, 550.*
320. M. Zorn, *Theorie der alternativen Ringe*, Abh. Math. Semin. Hamburg. Univ. **8** (1930), 123–147. *454, 509.*
321. ———, *Alternativkörper und quadratische Systeme*, Abh. Math. Semin. Hamburg. Univ. **9** (1933), 395–402. *509.*

Index

absolute Galois group, 279
absolutely simple group, 364
additive group, 326
adjoint anti-automorphism, 1
adjoint group, 364
adjoint involution, 2, 24, 42
adjoint representation, 343
adjoint semisimple group, 356
admissible pair, 525
affine group scheme, 325
Albert algebra, 517
Albert form, 235
Albert quadratic space, 235
Albert's theorem, 236, 308
algebraic group, 338
algebraic group scheme, 326
algebraic torus, 333
Allen invariant, 570
alternating bilinear form, xvi
alternating hermitian form, 42
alternative algebra, 451
annihilator of an ideal, 8
Arason invariant, 436
Arf invariant, xix
Artin's theorem, 454
Artin-Schreier sequence, 414
associative bilinear form, 453
associator, 454
augmentation Hopf ideal, 327
automorphism of a quadratic pair, 159

base of a root system, 353
bilinear trace form, 132, 454
biquaternion algebra, 233
Brauer group, 4
Brauer-equivalent algebras, 4

canonical involution, 89, 94, 119
canonical quadratic pair on $\lambda^m A$, 123
canonical quadratic pair on tensor products, 63
Cartan subspace, 146
Cartan-Dieudonné theorem, 178
Cartier duality, 333
Cayley algebra, 459
Cayley parametrization, 201
Cayley-Dickson algebra, 458
Cayley-Dickson process, 459
central algebra, 451
central isogeny, 342
central simple L-algebra, 552
central simple algebra, 3
centralizer, 5
centroid, 451
character, 327
characteristic polynomial, xv
Clifford algebra, 87, 92
Clifford bimodule, 110
Clifford group of a quadratic pair, 178, 351
Clifford invariant, 481, 563
closed embedding, 327
closed subgroup, 327
cohomological invariant of algebraic group, 429
cohomologous cocycles, 384
commutative group scheme, 325
comodule, 343
comorphism, 325
composition algebra, 456
composition algebra of type ${}^1A_2$, 475
composition algebra of type G_2, 466
composition algebra of type A_1, 466
composition of quadratic spaces, 488
cone of dominant weights, 354
connected component, 331
connected group scheme, 330
connected groupoid, 360
connecting map, 385
constant group scheme, 332
continuous Γ-action, 279
coordinate algebra, 543
corestriction, 329
coroot, 353
crossed product construction, 397
cubic cyclic algebra, 302
cubic norm, 489
cubic norm structure, 520
cyclic algebra of degree n, 414
cyclic composition, 495

degree, 452
degree of a central simple algebra, 4
degree of an algebra with involution, 21
Δ-associate space, 499

derivation, 334
derivation of a twisted composition, 569
descent condition, 361
determinant of a form, xvi, 80
determinant of a hermitian form, 114
determinant of a quadratic form, xvii
determinant of an element, xv
determinant of an involution, 81
diagonalizable group scheme, 332
Dickson invariant, 157, 167
Dieudonné's theorem, 453
differential, 335
dimension of an algebraic group, 337
direct sum of root systems, 353
discriminant, 291
discriminant algebra, 128
discriminant class of a quadratic pair, 409
discriminant of a form, xvi, 80
discriminant of a hermitian form, 114
discriminant of a quadratic form, xvii
discriminant of an involution, 81
discriminant quadratic extension, 409
distinguished involution, 306
division algebra, 3
division cubic norm structure, 521
double centralizer theorem, 5
dual numbers, 334
dual root system, 353
Dynkin diagram, 353

equivalent cocycles, 384
equivalent quadratic forms, xix
étale algebra, 281
étale group scheme, 332
even Clifford algebra, 87
even Clifford group, 349
even hermitian form, 42
exact sequence of pointed sets, 385
exceptional Jordan algebra, 513
exchange involution, 21
extended Clifford group, 181, 352

factor group scheme, 340
fiber functor, 342
fiber product, 328
field of definition, 378
finite group scheme, 333
first Tits construction, 526
first Tits process, 526
flag, 280
formally real Jordan Algebra, 542
Freudenthal "×"-product, 519
Freudenthal adjoint, 519
Freudenthal algebra, 517
Freudenthal's "magic square", 540

G-invariant vector, 343
Galois G-algebra, 288
Galois S_n-closure, 291
Galois descent, 279
Γ-embedding, 360
Γ-extension, 361
Γ-group, 383
Γ-module, 383
general linear group, 326
generalized cyclic algebra, 191
generalized pfaffian, 103
generic element, 451
generic minimal polynomial, 452
generic norm, 452
generic trace, 452
Goldman element, 32
Grassmannian, 9
group of R-points, 326
group of orthogonal similitudes, 350
group of symplectic similitudes, 347
group of unitary similitudes, 346
group scheme homomorphism, 325
group scheme of isometries, 345
group scheme of similitudes, 345
group scheme over an étale algebra, 330
group-like elements, 327
groupoid, 170

hermitian form, 23, 42
highest weight, 375
Hilbert's Theorem 90, 392
Hopf algebra, 324
Hopf algebra homomorphism, 324
Hopf ideal, 325
Hurwitz algebra, 460
hyperbolic algebra, 75
hyperbolic form, 74
hyperbolic involution, 75
hyperbolic quadratic pair, 77
hyperbolic quadratic space, 77
hyperplane reflection, 157

idealizer, 9
idempotent, 466
image, 327
improper similitude, 157, 164
index of a central simple algebra, 4
index of an algebra with involution, 73
induced Γ-group, 390
induced torsor, 390
injective homomorphism, 339
inner automorphism, 2
Inner form of an algebraic group, 425
intersection of algebraic groups, 328
invariant bilinear form, 453
inverse image, 328
involution, 13
involution of orthogonal type, 16
involution of symplectic type, 16
involution of unitary type, 21
involution trace, 45
involutions of the first kind, 13

involutions of the second kind, 13
irreducible representation, 343
irreducible root system, 353
isogeny, 342
isometry, 153, 159
isometry of a space, 155
isomorphism of root systems, 353
isotropic algebra, 74
isotropic algebra with involution, 73
isotropic ideal, 72, 73
isotropic involution, 73

Jordan algebra, 513
Jordan algebra of a quadratic form, 514
Jordan product, 27

kernel, 328, 385
Kummer sequence, 413

left-invariant derivation, 334
Lie algebra of a group scheme, 334
Lie algebra of Jordan type, 569
Lie algebra of type D_4, 569
Lie invariant function, 452
Lie product, 27
local triality, 567
loop in algebraic group, 432

maximal subtorus, 356
minimal dominant weight, 354
minimal Tits algebra, 377
Moore determinant, 239
multiplication algebra, 451
multiplicative group, 326
multiplicative quadratic form, 454
multiplicative type, 332
multiplicity of a weight, 344
multiplier, 488, 490
multiplier map, 184
multiplier of a similitude, 153, 158

nondegenerate bilinear form, xvi
nondegenerate quadratic space, xvii
nonsingular bilinear form, xvi, 1
nonsingular cubic form, 474
nonsingular hermitian form, 23, 42
nonsingular quadratic space, xvii
norm of a vector space, 37
norm of an algebra, 36, 554
normal subgroup, 327
normalizer, 327

Okubo composition algebra, 475, 480
1-cocycle, 361, 384
opposite algebra, 3
order of a finite group scheme, 333
orientation, 207
orientation of a quadratic space, 171
oriented quadratic space, 171
orthogonal group, 348, 350
orthogonal ideal, 72
orthogonal idempotents, 63
orthogonal involution, 16

para-Cayley algebra, 464
para-Hurwitz algebra, 464
para-quadratic algebra, 464
para-quaternion algebra, 464
para-unit, 466
Peirce decomposition, 516
Petersson algebra, 466
pfaffian, 20
pfaffian adjoint, 275
pfaffian characteristic polynomial, 19
pfaffian norm, 19
pfaffian trace, 19
Pfister bilinear form, xvi
Pfister form of an involution, 304
Pfister quadratic form, xix
pointed quadratic space, 520
polar of a cubic form, 474
polar of a quadratic form, xvii
principal homogeneous set, 288, 388
product algebra, 416
product of group schemes, 328
profinite group, 383
projective general linear group, 345
projective orthogonal group, 350
projective similitude, 159
projective symplectic group, 347
projective unitary group, 346
proper isometry, 154, 164
proper similitude, 154, 157, 164, 167
pseudodiscriminant, xix

quadratic map, 489
quadratic norm, 489
quadratic pair, 55
quadratic trace, 517
quasisplit group, 379
quaternion algebra, 25
quaternion basis, 25

radical of a quadratic form, 348
reduced characteristic polynomial, 5
reduced dimension, 6
reduced Freudenthal algebra, 517, 520
reduced norm, 5
reduced root system, 352
reduced trace, 5
reduced unitary Whitehead group, 267
reduced Whitehead group, 253
reflection, 352
regular bilinear form, xvi
regular form, 348
regular local ring, 337
regular quadratic space, xvii
regular space, 348
representation, 342

restriction map, 385
restriction scheme, 327
root, 353, 356
root lattice, 353
root system, 356, 364
roots of unity, 328
Rost invariant, 435

sandwich map, 32
Schur index, 4
second involution, 550
second Tits construction, 525
second Tits process, 526
semilinear action, 279
semisimple algebraic group, 355
separability idempotent, 285
separable algebra, 344, 453
separable homomorphism, 341
Severi-Brauer variety, 10
Shapiro's Lemma, 394
signature of a form, 136
signature of an involution, 137
similar spaces, 168
similitude, 488, 490
similitude of a bilinear space, 153
similitude of a quadratic pair, 159
similitude of an algebra with involution, 158
simple group, 357
simply connected group, 364
simply connected semisimple group, 356
simply transitive action, 388
skew field, 3
skew-hermitian form, 42
skew-symmetric bilinear form, xvi
Skolem-Noether theorem, 5
smooth algebraic group, 338
smooth group scheme, 338
SN-condition, 344
solvable algebraic group, 355
special Clifford group, 177
special Jordan algebra, 513
special linear group, 328
special unitary group, 346
spin group, 187
spinor group, 349, 351
spinor norm, 351
spinor norms, 187
split algebra, 4
split Cayley algebra, 462
split Freudenthal algebra, 517
split semisimple group, 356
split torus, 333
split trialitarian algebra, 553
split twisted composition, 490
splitting field, 4
Springer construction, 523
Springer decomposition, 523
stabilizer, 327
standard identification, 55
$*$-action, 376
strictly power-associative algebra, 451
sub-Grassmannian variety, 12
surjective homomorphism, 327
symbol algebra, 27
symmetric bilinear form, xvi
symmetric composition algebra, 464
symmetric composition of type ${}^iA_2'$, 480
symplectic basis, 16
symplectic group, 347
symplectic involution, 16
system of simple roots, 353

τ' (second involution), 550
Tits algebra, 377
Tits class of a simply connected semisimple group, 426
Tits class of an algebra with unitary involution, 427
Tits construction, 525
Tits process, 525
torsor, 288, 388
trace of an element, xv
transfer of a bilinear form, 524
transpose of a linear map, 1
trialitarian algebra, 552, 553
trialitarian algebra of type G_2, 553
trialitarian algebra of type ${}^1D_4$, 558
trialitarian algebra of type ${}^6D_4$, 558
trialitarian algebra of type ${}^3D_4$, 558
trialitarian algebra of type ${}^2D_4$, 558
trialitarian isomorphism, 553
trialitarian triple, 548
triality, 451, 547
trivial group, 326
twisted ρ-form, 392
twisted Cayley composition, 490
twisted composition of type G_2, 499
twisted composition of type ${}^1A_2'$, 505
twisted composition of type ${}^2A_2$, 506
twisted composition over L, 489
twisted form, 361
twisted Hurwitz composition, 499
twisting, 387
2-cocycle, 384
type of a root system, 356

unital algebra, 451
unitary group, 346
unitary involution, 21
unitary Whitehead group, 267

vector representation, 177

Wedderburn's theorem, 3, 303
weight, 344
weight lattice, 353
Weyl group, 353

Zorn matrices, 507

Notation

$()^{\#}$ (Freudenthal adjoint), 470, 519
$\times$ (Freudenthal product), 470, 519
$\bullet$ (Jordan product), 513
$\perp$, xvii
$\otimes = \otimes_F$ (tensor product), xix
$(C, n)_\Delta$, 500
(L, a) (cyclic algebra), 302, 414
$(U, q)_K$, 499
(V, Φ) (root system), 353
(V, q) (quadratic space), 74
(V, q, ζ) (oriented quadratic space), 171
$(Z, \alpha)_F$ (quaternion algebra), 190
(σ, f) (quadratic pair), 55
$(a, b)_F$ (quaternion algebra), 25
$(a, b)_{\zeta, K}$ (symbol algebra), 27
$\langle\langle \alpha_1, \dots, \alpha_n \rangle\rangle$ (Pfister bilinear form), xvi
$\langle\langle \alpha_1, \dots, \alpha_n]]$, xix
$\langle \alpha_1, \dots, \alpha_n \rangle$, xix
$\langle f, \xi \rangle$, 282
$[\alpha_1]$, xix
$[\alpha_1, \alpha_2]$, xix
$[a, b)_F$ (quaternion algebra in char. 2), 25
$\{a, b\}_i$, 452

$\mathsf{A}(\rho, w)$ (twisted ρ-forms of w), 362, 392
$\widetilde{\mathsf{A}}(\rho, w)$, 362
A^+ (Jordan algebra of A), 27, 513
A^0 (trace zero elements in A), 28, 472
$^{\mathbf{1}}\mathbf{A}$, 472
$^1A_2$, 475
$^1\mathsf{A}_3$, 227
$^1\mathsf{A}'_n$ (central simple algebras), 471
$^{\mathbf{2}}\mathbf{A}$, 478
$^2\mathsf{A}_2$, 480
$\underline{\mathsf{A}}_1$, 207
$\overline{\mathsf{A}}_1$, 464
A_1^2, 207
A_3 (alternating group), 421
A_n (root system), 354
A_n, 207
A_n^+, 515
$\underline{\mathsf{A}}^n$, 366
$\overline{\mathsf{A}}^n$, 366
A_α (twisted action), 387
A_χ (minimal Tits algebra), 426
a^χ, 431

Ad (adjoint representation), 343
ad (inner derivation), 344
A^Γ (Γ-invariant elements), 383
A_λ (Tits algebra), 377
Alg_F, 280
α_A^ϕ, 549
α_S, 481
$\alpha_{\sigma,\tau}$, 386
$\mathrm{Alt}(A, \sigma)$ (alternating elements in A), 14
A^{op} (opposite algebra of A), 3
A_{red}, 331
$(A^\times \times A^\times)^{\mathrm{Det}}$, 529
A–Tors_Γ (A-torsors), 388
$\mathbf{Aut}(A, \sigma)$, 345
$\mathbf{Aut}(J, A)$, 529
$\mathbf{Aut}(J/A)$, 529
$\mathrm{Aut}_{\widetilde{\mathsf{A}}}$, 362
$\mathbf{Aut}_{\mathrm{alg}}(A)$, 328
$\mathrm{Aut}_F(A, \sigma, f)$, 159
$\mathrm{Aut}_F(J, A)$, 529
$\mathrm{Aut}_F(J/A)$, 529
$\mathbf{Aut}_G(v)$, 328
$\mathbf{Aut}_K(A, \sigma)$, 345
$\mathrm{Aut}_R(A_R)$, 328

$B(A, \sigma, f)$ (Clifford bimodule of A), 110
$B(\theta)$ (Clifford bimodule functor), 175
B_n (root system), 354
B_n, 207
B'_n, 170, 369
B_n^+, 515
B_n, 172
$\underline{\mathsf{B}}^n$, 368
$\overline{\mathsf{B}}^n$, 368
B/A (left cosets), 385
$\beta(v, w)$, 522
$\beta_{\mathfrak{C}}$, 490
β_S, 490
$\hat{b}$, xvi
$b_{K/F}$, 534
b_q (polar form of q), xvii
$\mathrm{Br}(F)$ (Brauer group of F), 4
$b * (\ell_1 \otimes \ell_2)$, 311

$^{\mathbf{2}}\mathbf{C}$, 478
$C(-)$ (Clifford algebra functor), 173
$C(A, \sigma)$, 93, 175

$C(A,\sigma,f)$ (even Clifford algebra of A), 92
$C(G)$ (center of G), 357
$c(T)$ (Clifford invariant), 563
$C(V,q)$ (Clifford algebra of (V,q)), 87
$C_0(V,q)$ (even Clifford algebra of (V,q)), 87
${}^1\mathbf{C}$, 472
C_3 (cyclic group), 302
C_n (root system), 354
C_n, 207
C_n^+, 515
C^n, 369
$\overline{\mathsf{C}}^n$, 369
$C^+(V,q)$, 359
$C^-(V,q)$, 359
C_AB (centralizer of B in A), 5
$\mathfrak{C}_s$ (split Cayley algebra), 462
$\widetilde{\mathfrak{C}}$, 490
$CD(B,\lambda)$ (Cayley-Dickson process), 458
Comp_m (composition algebras), 458
Comp_m^+ (unital composition algebras), 458
$C^\perp$, 454
C_φ (Petersson algebra), 466
$\mathsf{Cubjord}$ (cubic norm structures), 520

$D(B,\tau)$ (discriminant algebra of (B,τ)), 128
${}^1\mathsf{D}_3$, 227
${}^1D_4$, 558
${}^2D_4$, 558
${}^3D_4$, 558
${}^6D_4$, 558
D_4, 562
D^4, 562
$\overline{\mathsf{D}}^4$, 562
D_n (root system), 355
D_n, 207
D_n^+, 515
D^n, 370
$\overline{\mathsf{D}}^n$, 370
$\deg A$ (degree of A), 4
Δ (Dickson invariant), 167
$\Delta(L)$, 291
$\Delta(X)$, 291
δ^0 (connecting map), 385
δ^1 (connecting map), 386
$\Delta(g)$ (Dickson invariant of g), 157
$\mathrm{Der}(A,M)$ (derivation module), 334
$\mathrm{Der}(\Gamma)$, 569
$\mathrm{Der}(J)$, 569
$\mathrm{Der}(J/L)$, 569
$\mathrm{Der}(V,L,Q,\beta)$, 569
$\det(m)$ (determinant of m), xv
$\det b$ (determinant of b), xvi
$\det q$, xvii
$\det\sigma$ (determinant of σ), 81
df (differential of F), 335
$\dim G$ (dimension of G), 337
$\operatorname{disc} b$ (discriminant of b), xvi
$\operatorname{disc} q$, xvii
$\operatorname{disc}\sigma$ (discriminant of σ), 81
$\mathrm{Dyn}(\Phi)$ (Dynkin diagram), 353

$E(a)$, 557
$E(\mathfrak{L})$ (Allen invariant), 570
E_6 (root system), 355
E_7 (root system), 355
E_8 (root system), 355
$(e_{ij})_{1\le i,j\le n}$ (standard basis of M_n), 30
$e_n(\pi)$ (cohom. inv. of the Pfister form π), 413
End, 171
$\mathrm{End}(\widetilde{\mathfrak{C}})$ (split trialitarian algebra), 553
$\mathrm{End}(\Gamma)$, 555
Et_F (étale algebras), 281

$f_1(B,\tau)$ (cohomology class), 422
$f_1(L)$ (cohomology class), 421
F_4 (root system), 355
F_4, 372
F^4, 372
$f_3(B,\tau)$ (cohomology class), 423
$f_i(J)$ (cohomology class), 534
$f_n(q_n)$ (cohomological invariant), 462
$F[\mathbf{V}]$, 326
$F[G]$, 325
$F\langle H\rangle$, 332
F_{alg} (algebraic closure), 323
f_{alg}, 339
F_χ (field of definition of χ), 426
F_λ, 378
φ_q (standard identification for $b=b_q$), 58
f_q (quadratic pair of q), 92
F_{sep} (separable closure), 280
f^* (comorphism), 327
f^t (transpose of f), 1
F_v (completion of F at v), 449

$G(A,\sigma)$ (multipliers), 162
$G(R)$ (group of R-points), 326
G^0 (connected component of G), 331
$g_1(L)$ (cohomology class), 421
$G_1\times G_2$, 328
G_2, 373
$\underline{g_2}(B,\tau)$ (cohomology class), 420
$\overline{\mathsf{G}_2}$, 464
G^2, 373
G_2 (root system), 355
$g_3(J)$, 537
G_L (restriction scheme), 326
$G^A(R)$, 325
$\mathbf{G}_{\mathrm{a}}$ (additive group functor), 326
Γ (absolute Galois group), 279
$\mathbf{\Gamma}^+(V,q)$, 349
$\mathbf{\Gamma}(A,\sigma,f)$ (Clifford group), 351
$\Gamma(A,\sigma,f)$ (Clifford group), 178
$\Gamma(C,L)$, 499
$\Gamma^+(V,q)$ (special Clifford group), 177

Γ_λ, 490
$^\gamma\xi$, 281
G^D (Cartier dual), 333
G–Gal_F (Galois algebras), 290
$\mathbf{GL}(V)$ (general linear group), 326
$\mathbf{GL}_1(A)$, 326
$\mathbf{GL}_n$, 326
$\mathbf{G}_{\mathrm{m}}$ (multiplicative group functor), 326
$\mathbf{G}^1_{\mathrm{m},K}$ (norm one elements), 402
$\mathfrak{g}(A,\sigma)$, 28
$\mathbf{GO}(A,\sigma,f)$, 350
$\mathrm{GO}(A,\sigma)$, 159
$\mathbf{GO}(A,\sigma)$, 347
$\mathrm{GO}(A,\sigma,f)$, 159
$\mathrm{GO}(V,b)$, 154
$\mathrm{GO}(V,q)$, 155
$\mathrm{GO}^+(V,q)$, 157
$\mathrm{GO}^+(A,\sigma)$, 164
$\mathrm{GO}^+(A,\sigma,f)$, 167
$\mathrm{GO}^+(V,b)$, 154
$\mathrm{GO}^-(A,\sigma,f)$, 202
G^q (quasisplit inner form of G), 426
$\mathrm{Gr}(rn,A)$ (Grassmannian), 9
Groups, 325
$\mathrm{GSp}(A,\sigma)$, 159
$\mathrm{GSp}(V,b)$, 154
$\mathbf{GSp}(A,\sigma)$, 347
G^*, 327
G–Tors_Γ (G-torsors), 290
$\mathrm{GU}(A,\sigma)$, 159
$\mathrm{GU}(B,\tau)$, 193
$\mathbf{GU}(B,\tau)$, 346
$\mathrm{GU}(V,h)$, 155

$\mathcal{H}(A,\sigma)$ (Jordan algebra of $\mathrm{Sym}(A,\sigma)$), 28
$\mathcal{H}_n(C,\alpha)$, 516
$\mathcal{H}_3(C,\alpha)^0$, 517
$H^0(\Gamma,A)$, 383
$H^1(\Gamma,A)$ (cohomology set), 384
$H_1\cap H_2$, 328
$H^2(\Gamma,A)$ (cohomology group), 384
$H^i(F,A)$ (cohomology groups), 391
H_{const} (constant group scheme), 332
H_{diag} (diagonalizable group scheme), 332
H_{et}, 332
${}_nH$ (n-torsion of H), 413
$\mathrm{Hom}_F(G,H)$ (group scheme homom.), 325
Hurw_m (Hurwitz algebras), 460
$\overline{\mathit{Hurw}}$(para-Hurwitz algebras), 464

$i(G)$ (Rost invariant), 435
I^0 (annihilator of I), 8
IF (fundamental ideal of WF), 141
I^nF (powers of the fundamental ideal), 141
$\mathrm{im}(f)$ (image of F), 327
$\mathrm{Ind}_{\Gamma_0}^{\Gamma} A_0$ (induced Γ-group), 390
$\mathrm{ind}\,A$ (Schur index of A), 4
$\mathrm{Ind}_H^G M$, 288
$\mathrm{Int}(v)$ (inner automorphism), 2

$I^\perp$ (ideal orthogonal to I), 72
$\mathrm{Isom}(\mathsf{X})$, 170
$\mathrm{Iso}(A,\sigma)$, 159
$\mathrm{Iso}(V,b)$ (isometries of (V,b)), 153
$\mathbf{Iso}(A,\sigma)$ (isometries of (A,σ)), 345

$J(A,\lambda)$ (first Tits construction), 526
$J(B,\tau,u,\nu)$ (second Tits construction), 524
$J(L,V)$ (Springer construction), 523
$J(V,q)$ (Jordan algebra of a quadratic form), 514
J^0, 514

$K*K'$, 416
$\ker(f)$ (kernel of f), 328

L^0, 284
Λ (root lattice), 353
Λ_+ (cone of dominant weights), 353
Λ_r, 353
$\lambda^k a$, 195
L^G (invariant elements), 287
$\mathfrak{L}(A)$ (Lie algebra of A), 27
$\mathrm{Lie}(G)$ (Lie algebra of G), 334
$\mathfrak{L}(T)$, 569
L_K, 280
ℓ_x (left multiplication with x), 455

$M(A)$ (multiplication algebra), 451
$M_{[K]}$ (twisted module), 416
$M_n(C)$ (matrix algebra), 516
$M_{r,s}(D)$ ($r\times s$-matrices), 7
$\mu(t)$ (multiplier of t), 483
$\mu_i(a,b)$, 452
$\boldsymbol{\mu}_n$ (roots of unity), 328
$\boldsymbol{\mu}_{n[K]}$ (twisted group), 418
$\mu_n^{\otimes i}(F)$, 431
$\overline{\mu}$ (multiplier map), 182
$\underline{\mu}$ (multiplier map), 184

$N(\ell)$ (norm of ℓ), xvi
$N(x,y,z)$ (cubic form), 474
N_A (generic norm), 452
N_C (norm of x), 457
$\mathrm{nil}(A)$ (nilpotent elements), 331
N_J (norm of J), 516
$N_{K/F}(A)$ (norm (corestriction) of A), 37
$N(K/F)$ (group of norms), 114
$N_{L/F}(\ell)$ (norm of ℓ), xvi
$\mathrm{Nrd}_A(a)$ (reduced norm of a), 5
$\mathbf{N}_U$, 327

$\mathbf{O}(A,\sigma)$, 347
$\mathrm{O}(A,\sigma,f)$, 160
$\mathbf{O}(A,\sigma,f)$, 350
$\mathrm{O}(V,q)$, 155
$\mathbf{O}(V,q)$, 348
$\mathrm{O}(A,\sigma)$, 159
$\mathrm{O}^+(A,\sigma)$, 164
$\mathbf{O}^+(A,\sigma,f)$, 351

$\mathrm{O}^+(V,b)$, 154
$\mathrm{O}^+(V,q)$, 157
$\mathbf{O}^+(V,q)$, 348
$\mathrm{O}(V,b)$, 154
Oku, 480
$^1\mathsf{Oku}$, 475
$^2\mathsf{Oku}$, 480
$\Omega(A,\sigma)$, 181
$\Omega(A,\sigma,f)$ (extended Clifford group), 181
$\boldsymbol{\Omega}(A,\sigma,f)$ (extended Clifford group), 352

$P(R)$ (fiber functor), 342
$\wp$ (Artin-Schreier map), xviii, 414
$P_{A,x}(X)$ (generic polynomial), 452
$\mathbf{PGL}(V)$, 345
$\mathbf{PGL}_1(A)$, 344
$\mathbf{PGL}_n$ (projective linear group), 345
$\mathrm{PGO}(A,\sigma)$, 159
$\mathrm{PGO}(A,\sigma,f)$, 159
$\mathbf{PGO}(A,\sigma)$, 347
$\mathbf{PGO}(A,\sigma,f)$, 350
$\mathbf{PGO}^+(A,\sigma,f)$, 351
$\mathrm{PGO}(V,b)$, 154
$\mathbf{PGO}(V,q)$, 350
$\mathrm{PGO}^+(A,\sigma,f)$, 167
$\mathrm{PGO}^+(V,b)$, 154
$\mathrm{PGSp}(A,\sigma)$, 159
$\mathbf{PGSp}(A,\sigma)$, 347
$\mathrm{PGSp}(V,b)$, 154
$\mathrm{PGU}(A,\sigma)$, 159
$\mathbf{PGU}(B,\tau)$, 346
$\mathrm{PGU}(V,h)$, 155
$\pi(\tau)$ (Pfister form of an involution), 304
$\pi_0(A)$, 330
$\mathrm{Pic}(G)$ (Picard group), 431
$P_{J,a}(X)$ (generic minimal polynomial), 519
$P_{L,\ell}(X)$ (characteristic polynomial), xvi
$P_m(X)$ (characteristic polynomial), xv
$\mathrm{PGO}(V,q)$, 155
PQ_m (Pfister forms), 460
$\mathrm{Prd}_{A,a}(X)$ (reduced characteristic polynomial of a), 5
$\mathrm{Prp}_{\sigma,s}(X)$ (pfaffian characteristic polynomial of s), 19
p_σ (pfaffian adjoint), 245
$\mathrm{PSim}(A,\sigma)$, 159
$\mathrm{PSim}(V,b)$ (projective similitudes of (V,b)), 153
p_θ, 246

$Q^+(A,\sigma,\ell)$, 411
$Q(A,\sigma)$, 411
$Q_0(A,\sigma,\ell)$, 411
q_b, xvii
Q_n, 171
Q_n^1, 171
Q_τ^0, 306
$\mathrm{Quot}(S)$ (field of fractions of S), 341
$\mathbb{Q}/\mathbb{Z}\,(i)(F)$, 431

$R(C)$, 455
$\mathrm{rdim}\,M$ (reduced dimension of M), 6
res (restriction map), 385
ρ_v (hyperplane reflection), 157
${}^\rho(E\otimes\Delta,\sigma\otimes 1)$, 552
${}^{\rho^{-1}}\alpha_{\rho E}$, 554
ρ_R, 325
$R_{L/F}(G)$ (corestriction functor), 329
$R_{L/F}(G)$ (corestriction of G), 329

S, 187
$S(\ell)$, xvi
$S(V^*)$ (symmetric algebra), 326
$\underline{S^0}$, 284
$\overline{\mathsf{S}_2}$, 464
$S_{L/F}$, 283
S_A, 469
$\mathrm{Sand}(x)$ (sandwich map), 32
$\mathrm{SB}_r(A)$ (Severi-Brauer variety of A), 10
Scomp_m (symmetric composition algebras), 464
$^2\mathsf{Sepalt}_m(3)$ (cubic separable algebras), 478
$\mathsf{Sepjord}_n(m)$ (separable Jordan algebras), 514
Sets_Γ (Γ-sets), 281
$\mathrm{sgn}_P\,\sigma$ (signature of σ), 138
$\mathfrak{s}(B,\tau)$, 198
$\mathrm{SGU}(B,\tau)$, 194
$s_i(m)$ (coefficients of $P_m(X)$), xv
$\Sigma(L)$, 291
$\Sigma(X)$, 290
σ_b (involution adjoint to b), 1
σ_g (involution), 13
σ_q (adjoint involution of q), 58
$\Sigma_\sigma(A)$, 266
σ_*, 18
$\underline{\sigma}$ (induced involution on $C(A,\sigma,f)$), 94
$\mathrm{Sim}(A,\sigma)$ (similitudes of (A,σ)), 158
$\mathbf{Sim}(A,\sigma)$ (similitudes of (A,σ)), 345
$\mathrm{Sim}(V,b)$ (similitudes of (V,b)), 153
$S_J(a)$, 516, 519
$SK_1\,\mathrm{U}(B)$ (reduced unitary Whitehead group), 267
$\mathrm{SK}_1(A)$ (reduced Whitehead group), 253
$\mathrm{Skew}(A,\sigma)$ (skew-symmetric elements in A), 14
$\mathrm{Skew}(A,\sigma)^0$ (trace zero elements), 28
$\mathrm{SL}_1(A)$, 253
$\mathrm{SL}(E)$, 194
$S_{L/F}(\ell)$, xvi
$\mathbf{SL}(V)$, 328
$\mathbf{SL}_1(A)$, 328
Sn (spinor norm), 186
$\mathrm{Sn}(A,\sigma,f)$, 187
$\mathrm{Sp}(A,\sigma)$, 159
$\mathbf{Sp}(A,\sigma)$, 347
$\mathrm{Sp}(V,b)$, 154
$\mathbf{Sp}(V,h)$, 347

$\mathrm{Spin}(A,\sigma,f)$, 187
$\mathbf{Spin}(A,\sigma,f)$, 351
$\mathrm{Spin}(V,q)$, 187
$\mathbf{Spin}(V,q)$, 349
$\mathbf{Spin}_8$, 486
$\mathbf{Spin}^{\pm}(V,q)$, 359
$\mathrm{Srd}_A(a)$, 5
$\mathrm{SSym}(A,\sigma)^{\times}$, 406
$\mathrm{SSym}(B,\tau)^{\times}$, 531
$\mathrm{SSym}(B,\tau)^{\times}$, 401
$\mathrm{SU}(B,\tau)$, 194
$\mathbf{SU}(B,\tau)$, 346
$\mathrm{SU}(V,h)$, 155
$\mathbf{S}_v$, 327
$\mathrm{Sym}(A,\sigma)$ (symmetric elements in A), 14
$\mathrm{Sym}(A,\sigma)'$, 400
$\mathrm{Sym}(B,\tau)^0$, 306
$\mathrm{Symd}(A,\sigma)$ (symmetrized elements in A), 14

$T(\ell)$ (trace of ℓ), xvi
$T(A,\sigma)$ (trialitarian algebra), 558
$t(B,\tau)$ (Tits class), 427
$T(\mathfrak{L})$ (trialitarian algebra of a Lie algebra), 570
$T(x,y)$ (bilinear trace form), 281
t^+, 483
t^-, 483
T_A (generic trace), 452
τ', 550
$T_C(x)$ (trace of x), 457
θ^+, 485
θ^-, 485
θ_α, 387
$T_J(a)$, 516, 519
$T_{L/F}(\ell)$ (trace of ℓ), xvi
$(T_{L/F})_*(b)$, 524
$\mathrm{tr}(m)$ (trace of m), xv
$\mathrm{Trd}_A(a)$ (reduced trace of a), 5
T_τ, 534
$T_{\tau,\tau'}(x,y)$, 534

$\mathbf{U}(B,\tau)$, 346
$\mathrm{UK}_1(B)$ (unitary Whitehead group), 267
$\mathrm{U}(A,\sigma)$, 159
$\mathrm{U}(B,\tau)$, 193
$\mathrm{U}(V,h)$, 155

$V^{\perp}$, xvii
V^* (dual space of V), xvi
V^G (G-invariant subspace), 343

$W(\Phi)$ (Weyl group), 353
WF (Witt ring of F), 141

$x^{\#}$, 470, 519
$X(f)$, 281
$X(L)$, 280
$x \star y$, 472
X_c, 444
x^f (left module homomorphism), xv
$\chi(\lambda)$, 354
χ_i, 357
$x *_\sigma (a \otimes b)$ (right module structure), 32

$Z(A)$ (centroid), 451
$Z(A,\sigma,f)$, 187
$Z^1(\Gamma,A)$ (1-cocycles), 384
$Z^2(\Gamma,A)$ (2-cocycles), 384