

# Algebraic $K$ -Theory, Linear Algebraic Groups, and Related Structures

RTN Network HPRN-CT-2002-00287

Periodic Progress Report 1.10.2003 – 30.09.2004

Network Full Title: Algebraic  $K$ -Theory,  
Linear Algebraic Groups, and Related Structures

Network Short Title:  $K$ -Theory and Algebraic Groups

Network Home Page: <http://www.mathematik.uni-bielefeld.de/K+AG/>

Commencement Date: 1.10.2002

Duration of Contract: 48 months

Period covered by this report: 1.10.2003 – 30.09.2004

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## Part A – Research Results

### A.1 Scientific Highlights

- The network is proud of having been selected to be represented by a survey talk at the 4ECM (Fourth European Congress of Mathematics, Stockholm, June 27 – July 2, 2004 <sup>1</sup>), One of the postdoctoral scientists of the network, Marina Monsurro (Team 9: Lausanne), has given that representation<sup>2</sup>.
- The former postdoc of our network and up to now local coordinator of our Regensburg team (Team 2), Susanne Pumplün, obtained a permanent position at the university of Nottingham, UK
- Ahmed Laghribi from Casablanca, Morocco), who was working in the Bielefeld network node, first as an Alexander von Humboldt fellow, later as a RTN postdoctoral scientist, obtained a permanent position at the university of Lens.
- Kirill Zainoulline from St. Petersburg (Team 13) University (Team 1) obtained a five months extension of his Alexander v. Humboldt Fellowship, he will continue to work in the Bielefeld Team 1.
- Detlev Hoffmann, up to now our local coordinator at Besançon, accepted a professorship at Nottingham University, UK.
- A major scientific event with huge impact in training was a special semester on “K-Theory and Noncommutative Geometry” given at Paris 7 (Jussieu). It was organized by Max Karoubi and Ryszard Nest and focussed on Noncommutative Geometry and Algebraic K-theory, as well as the interrelation between the two subjects.

During the semester three long courses and five short courses were given by international lectures on topics like

- Formes Modulaires et Géométrie Non Commutative
- Introduction to motivic homotopy theory of schemes
- Cyclic homology, noncommutative differential calculus and index theorems
- Cohomology and K-theory
- The de Rham-Witt complex and topological Hochschild homology
- KK-theory and applications
- Deformation quantization and index theorems
- K-theory and cyclic homology
- Operads and formality theorems

A series of lectures on advanced topics, and several workshops, seminars and discussion groups were organized.

One of the main activities of this semester was a two weeks international conference at Paris 7:

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<sup>1</sup><http://www.math.kth.se/4ecm/>

<sup>2</sup><http://www.math.kth.se/4ecm/program/european.nw.lectures.html>

- K-Theory and Noncommutative Geometry, Clay Mathematics Institute Conference, sponsored by the Leibniz prize of Joachim Cuntz, the Mathematical Institute of Jussieu and the European Community (this Network) 5-17 July 2004

During this conference, 48 talks were given by international speakers.

The following workshops and conferences have been held during the reporting period and are to be considered as scientific highlights as well. All these events are documented in more detail on the home page of our network: <http://www.math.uni-bielefeld.de/K+AG/>

- February 1 – 5, 2004, Eilat: Workshop on Linear Algebraic Groups, Quadratic Forms and Related Topics. 43 international scientists attended this workshop, among them 19 from this network.
- March 1 – July 17, 2004, Paris, Institut Henri Poincare, Semester on K-Theory and Noncommutative Geometry (see above), organized by Max Karoubi, Ryszard Nest, Claude Lucet.
- Mai 12-15, 2004 Nottingham: LMS Meeting and Workshop, organized by Detlev Hoffmann and John Cremona, with 13 talks by invited speakers, mostly from this network, as well as several contributed talks by young researchers of this network.
- June 21 – 25, 2004, Mini-cours, Lens, Juin 2004 “Formes Quadratiques”: Five international experts were invited to give lectures on various issues around the main topic. 65 participants, including many young researchers, attended this workshop.
- June 27 – July 2, 2004, 4ECM: Fourth European Congress of Mathematics. This network was represented by a talk given by Marina Monsurro, one of this network’s “young scientists”, see above.
- June 28 – July 2, 2004, Sestri Levante (Genova): Summer School and Workshop Motives, K-Theory and Arithmetical Geometry The School has mainly been intended for young researchers coming from European Universities participating in the Network. International senior experts have given several series of lectures on Motivic Galois group, Chow groups of varieties over finite fields and p-adic fields, Categories of motives, Birational motives, Algebraic Stacks, Motivic cohomology and Bloch-Kato Conjecture, and there have also been 12 research talks given by the participants, including several young researchers coming from the teams of the Network. The lecture notes, provided by the speakers, have been posted on the Web site <http://www.mathematik.uni-bielefeld.de/LAG/>
- July 5 – 8, 2004, Dublin 2004 Workshop on K-Theory, Algebraic Groups and Related Structures:

The workshop involved nearly 40 researchers, including a substantial number of young researchers from different nodes of the network.

In the mornings there were three series of talks by international experts on the topics of "K-theory and number theory", "Algebras with involution", and "Algebraic groups" respectively. These were of particular value and benefit to the young researchers especially. There was also one hour-long talk each evening by an international expert, and there were a number of shorter talks, many given by the young researchers themselves.

- July 5 – 17, 2004, Paris, Clay Mathematics Institute Conference on K-Theory and Noncommutative Geometry, organized by P. Baum, E. Blanchard, J. Cuntz, B. Kahn, M. Karoubi, R. Nest, C. Weibel.

All these workshops have been attended by many young researchers from almost all other teams and had a considerable training effect.

- Many research projects mentioned in the workplan have been successfully approached, which is documented by numerous publications which are not "Joint Publications" in the sense of this network – and therefore are not listed here – but also by the "Joint Publications" as listed below.

## A.2 Joint Publications

*Manuscripts with at least one of our young researchers as an author have been marked by (Y).*

Teams 9+1: (Y) Giordano Favi, Emmanuel Lequeu: A criterion for a central simple algebra to be split (8 p.)

<http://www.math.uni-bielefeld.de/LAG/>, nr 173.

Teams 10+11: Boris E. Kunyavskii, Louis H. Rowen, Sergey V. Tikhonov, and Vyacheslav I. Yanchevskii: Division algebras that ramify only on a plane quartic curve <http://www.mathematik.uni-bielefeld.de/LAG/>, nr 161.

Teams 1+4: Jurgen Hurrelbrink, Nikita A. Karpenko, and Ulf Rehmann: The Minimal Height of Quadratic Forms of Given Dimension (8 p.) submitted

<http://www.mathematik.uni-bielefeld.de/LAG/>, nr 157.

Teams 5+11: D.F. Bazyleu, J. Van Geel, V. I. Yanchevskii: Omega-algebras over Henselian Discrete Valued Fields with real closed residue field (24 p.)

<http://www.mathematik.uni-bielefeld.de/LAG/>, nr 156.

Teams 5+11: Sergey V. Tikhonov, Jan Van Geel, Vyacheslav I. Yanchevskii: Pythagoras numbers of function fields of hyperelliptic curves with good reduction (17 p.)

<http://www.mathematik.uni-bielefeld.de/LAG/>, nr 155.

Teams 9+13: M.Ojanguren, I.Panin, K.Zainoulline: On the Norm Principle for Quadratic Forms (14 p.)

<http://www.mathematik.uni-bielefeld.de/LAG/>, nr 152.

Teams 1+2: (Y) Baptiste Calmès and Jens Hornbostel: Witt Motives, Transfers and Reductive Groups (29 p.)

<http://www.mathematik.uni-bielefeld.de/LAG/>, nr 143.

Teams 13+5: Louis Rowen, Alexandre Sivatski, Jean-Pierre Tignol: Division Algebras Over Rational Function Fields in One Variable (24 p.)

<http://www.math.uni-bielefeld.de/LAG/>, nr 137.

Teams 6+5: (Y) David W. Lewis, Thomas Unger and Jan Van Geel: The Hasse principle for similarity of hermitian forms (20 p.)

<http://www.math.uni-bielefeld.de/LAG/>, nr 130.

Teams 3+1: (Y) Detlev W. Hoffmann and Ahmed Laghribi: Isotropy of quadratic forms over the function field of a quadric in characteristic 2 (29 p.)

<http://www.math.uni-bielefeld.de/LAG/>, nr 129.

Teams 4+12: (Book, accepted for publication) V. Franjou, E. Friedlander, T. Pirashvili and L. Schwartz. Polynomial functors, Steenrod algebra and the cohomology of finite group schemes. Panoramas et Synthèses. Soc. Math. de France.

Teams 2+13: (Y) Hornbostel, J., Yagunov, S. Rigidity for henselian local rings and A1-representable theories. Preprint

(<http://www.math.uiuc.edu/K-theory/0688>)

Teams 1+13: Nenashev, A., Zainoulline, K. Oriented Cohomology and Motivic Decompositions of Relative Cellular Spaces. Preprint 2004, 23pp

(<http://www.math.uiuc.edu/K-theory>).

Teams 9+13: Ojanguren, M., Panin, I., Zainoulline, K. On Norm Principle for Quadratic Forms. Submitted to Journal of Ramanujan Math. Soc. 2004.

Teams 6+11: edrini and V. Guletskii :Finite dimensional Motives and the Conjectures of Beilinson and Murre, K-Theory 30, 243-263 (2003)

Teams 4+6: B.Kahn, J.Murre and C.Pedrini : On the Chow Motive of an algebraic surface ,preprint,Sept 2004.

Teams 4+6: B.Kahn and L.Barbieri-Viale : 1-Motives (preprint July 2004)

Teams 4+6: Contemporary Developments in Algebraic K-Theory : M.Karoubi,A.Kuku and C.Pedrini Editors, ICTP Lectures Notes (2003)

Teams 5+6: K-Theory ,A Special Issue in Honor of H.Bass: M.Karoubi,A.Kuku and C.Pedrini Editors ,K-Theory (2003)

Teams 3+5: Anne Cortella and Jean-Pierre Tignol, Skolem-Noether pour les algèbres d'endomorphismes de modules de torsion sur des anneaux principaux, submitted.

## Part B – Comparison with the Joint Programm of Work (Annex I of the Contract)

### B.1 Research Objectives

The research objectives, as set down in the Work Programme (Annex I of the contract) are still relevant and achievable.

### B.2 Research Method

No change of the research method turned out to be necessary.

### B.3 Workplan

All the events scheduled in the workplan until September 2004 have been taken place<sup>3</sup>, except for the summerschool planned at Edinburgh, which was postponed to 2005.

The workplan for the next reporting period is scheduled as foreseen in Annex I of the contract, except that the workshop planned by Regensburg might actually take place at Nottingham, due to the changes of the network as a whole.

Professional research effort on the network project (2 <sup>nd</sup> reporting period)			
Participant	Young researchers to be financed by the contract (person-months)	Researchers to be financed from other sources (person-months)	Researchers likely to contribute to the project (number of individuals)
	(a)	(b)	(c)
1. Bielefeld	17	36	18
2. Regensburg	25	15	9
3. Besancon	14	36	18
4. UP7	13	36	17
5. UCL	9	36	17
6. DIMA	5	24	12
7. UEDIN	4	21	9
8. NUID/UCD	6	21	9
9. EPFL	8	57	22
10. Bar-Ilan	0	18	10
11. Minsk	0	6	3
12. RMI	0	14	7
13. RAS	0	10	5
Totals	101	330	156

<sup>3</sup>even those which were scheduled before the actual start of the network on Sept. 1, 2002

## **B.4 Organisation and Management**

### **B.4.1**

The network is organized along the rules which have been set up in the work programme section 4. Various visits for collaboration of single scientists at other nodes have been undertaken.

Decisions relevant for the network were made on the basis of email discussions and by personal contact during the network events.

Dissemination and communication of research results were done by the network's preprint server.

This server published, since the beginning of the reporting period, 53 manuscripts dealing with topics closely related to the network research, written by authors from all over the world, plus 6 Lectures given at the Network Conference: Motives, K-theory and Arithmetical Geometry Summer School & Workshop, Sestri Levante (Genova, Italy) June 28 - July 2, 2004.

The server is already accepted as a dissemination and publication basis by scientists worldwide working in the area of this network.

### **B.4.2**

The relevant network meetings have been listed in section A.1, as they are considered as highlights.

C. Pedrini and Luca Barbieri-Viale have represented the Network at the EAGER Conference 2004 "Workshop Algebraic Cycles and Motives" (Leiden, The Netherlands) from 30 Aug 2004 to 3 Sep 2004. L.Barbieri-Viale has given an invited talk on "Motivic Albanese".

C. Pedrini has represented the Network at the International Conference : "Workshop on Algebraic Geometry and Physics in Lisboa (Portugal) from Sept 6 2004 to Sept 12 2004, where he gave an invited talk on "K-theory and the theory of Motives"

### **B.4.3**

Tabular description of the networking which has taken place during the reporting period:

From/To	1	2	3	4	5	6	7	8	9	10	11	12	13
1 Bielefeld		p	p	p	5	v		v	p	v			p
2 Regensburg	p			v				vp		v		p	p
3 Besançon	vp	v		v	cp					c		c	
4 U Paris 7	p	v			v	p				v		p	
5 Louvain	p	v	pc	v		p		p	p	v	pc		p
6 Genova				p	p			v		v	p		
7 Edinburgh				v						c			
8 Dublin		vp		v	p			v		v			
9 Lausanne	p	v		v	p			v					pc
10 Bar-Ilan	vc		v	v	vc		c	v			p	v	
11 Minsk				v	pv	p		v		p			
12 Tbilisi	vps			p	v					c			
13 St. Petersburg	vps	p		v	p			v	pc	v			

*p=joint publication,*

*v=Visit(e.g. for workshop or seminar),*

*s=secondment (possibly financed by other grants like Humboldt Fellowships or Prizes)*

*y=young scientist secondment*

*c=active cooperations (in the sense of preparing publications, joint seminars)*

Beyond these mentioned direct cooperations between the nodes, there were many other cooperations. For example, during the reporting period there was a workshop at Nottingham, England, on Quadratic Forms and related topics, in May 2004, at which scientists from many nodes actively participated (see A.1).

## B.5 Training

### B.5.1 Advertisement of open positions on various web pages:

- own network's home page:  
<http://www.mathematik.uni-bielefeld.de/K+AG/>
- befriended international networks' home pages and others like:  
<http://www.math.uiuc.edu/K-theory/>  
<http://www.lehigh.edu/~dmd1/algtop.html>
- but also on the home pages of the universities and departments involved.

As the tables below and in Part C show, more than 50% of the deliverable person-months have already been committed now.

### B.5.2 Table to compare progress in recruitment of young researchers with the plan of the contract:

The table shows that 101 person-months of post- and predoc training have been given during this reporting period.

At several sites, additional training from other sources was given as well (e.g., Marie Curie and A.v.Humboldt fellowships) in the context of the network research topics. The table in part C shows that, more than half of the deliverable person-months have already been contracted (179 out of 320).

Participant	Contract deliverable of Young Researchers to be financed by the contract (person-months)			Young Researchers financed by the contract so far (person-months)		
	Pre-doc (a)	Post-doc (b)	Total (a+b)	Pre-doc (c)	Post-doc (d)	Total (c+d)
Bielefeld		44	44		31	31
Regensburg		33	33	12	20	32
Besançon		53	53	11	15	26
U Paris 7		28	28		13	13
Louvain(UCL)		36	36	8	24	32
Genova(DIMA)		36	36	2	12	14
Edinburgh(UEDIN)		15	15		7	7
Dublin(NUID/UCD)		15	15		12	12
Lausanne(EPFL)		30	30		12	12
Bar-Ilan		30	30		0	0
Minsk		0	0		0	0
Tbilisi		0	0		0	0
St. Petersburg		0	0		0	0
<b>TOTAL</b>	<b>0</b>	<b>320</b>	<b>320</b>	<b>33</b>	<b>129</b>	<b>179</b>

**B.5.3, B.5.4** The young researchers have been integrated to the research seminars given at the universities, as well as other research activities like local conferences and workshops. They were given the opportunity to participate in the workshops organized by the network, in which particular survey and introductory courses into various special subfields of the network's research objectives were given by experienced and senior scientists.

### **B.6 Difficulties**

The coordinator of the Besançon team, Detlev Hoffmann, has moved to the University of Nottingham, UK.

The coordinator of the Regensburg team, Susanne Pumplin, has accepted an offer of a permanent position at the University of Nottingham, UK.

Since these two are very active scientists in the areas of this network concerning both research and training, it makes sense to make Nottingham a new node of this network. Besançon is willing to split its network finances, and it may be that some of the finances from Regensburg can be transferred to Nottingham.

The network will propose a contract amendment containing these suggestions. All network partners already have agreed by writing to this change.

## **Part C – Summary Reports by Young Researchers**

A list of contracts with Young Researchers is given on the next page.

The two columns "months contracted" give the number of months in this reporting period (first column) and the number of months already contracted in total.

**In total the table shows that the network was able to contract more than half of their "deliverable" months.**

*(One line for each contract, first and last month of contract are given in the respective year column. In case of 1-months-contracts only this month is indicated.)*

Name	Participant	Months contracted		Begin/End of Contract					Report included
		this period	total so far	2002	2003	2004	2005	2006	
Avitabile	Regensburg	0	1		07				
Astier	Regensburg	3	4		09 12				
Astier	Regensburg	2	2			08 09			
Becher	Louvain	6	6			04 09			yes
Calmès	Bielefeld	6	6			03 08			yes
Calmès	Regensburg	5	5		09	02			yes
Calmès	Louvain	1	6			09	02		
Canepa	Genova	0	1		09 10				
Delaunay	Lausanne	4	4			04 07			
Delaunay	Lausanne	1	1			09	01		
Felisatti	Dublin	0	6	10	03				
Friedl	Edinburgh	1	1			04			yes
Frings	Louvain	0	12	10	09				
Garcia	Regensburg	0	1		08				
Garcia	Regensburg	1	1			07			yes
Gomez	Regensburg	1	1			07			yes
Guffroy	Genova	4	12			06	05		
Haesemeyer	Paris	2	2			06 07			yes
Hornbostel	Paris	2	2			04 05			yes
Laghribi	Bielefeld	11	19		02	08			
Lequeu	Bielefeld	0	6			10	03		
Mazza	Paris	8	8			01 08			yes
Nespolino	Louvain	0	6		03 08				
Nespolino	Besançon	11	11		10	08			
Ntokas	Besançon	3	15	10	12				
Paugam	Regensburg	10	11		09	07			yes
Renaudin	Edinburgh	3	6		07 12				
Schlichting	Paris	1	1			06			yes
Sixt	Louvain	2	2		11 12				
Unger	Regensburg	0	1		03				
Unger	Regensburg	0	1		06				
Unger	Regensburg	1	1			03			
Unger	Regensburg	1	1			06			yes
Velusek	Regensburg	1	1			06			yes
Villa	Dublin	6	6		10	03			
Wittmann	Lausanne	3	3		10 12				yes
<b>TOTAL</b>		<b>101</b>	<b>179</b>						

Reports by those scientists marked “yes” are appended.

## The Updated Database Report

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### **The Annual Cost Statements**

13 Cost statements are appended.