Reconstruction of hypersurface singularity from its triangulated Categ of Singularities

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$S = k[x_1, \cdots, x_d]$. char $k = 0$

$W \in S$ $\quad W = O(x^2)$

<u>Thm</u> (Orlov) $HMF(W) \cong D_{sg}(W^{-1}(0))$

$$(E^0 \xrightarrow{\delta^0} E^1 \xrightarrow{\delta^1} E^0) \longmapsto \mathrm{cok}(\delta^1)$$

$0 \in W^{-1}(0)$ - isolated sing

$\mathcal{O}_0 \in D_{sg}(W^{-1}(0))$ - generator

$\overline{D_{sg}(W^{-1}(0))} \cong \mathrm{perf}(R\mathrm{End}(\mathcal{O}_0))$

$R\mathrm{End}(\mathcal{O}_0) =: B_W$

dg End alg.

$\Lambda(V) \cong \mathrm{Ext}^{\cdot}_{D_{sg}(W^{-1}(0))}(\mathcal{O}_0, \mathcal{O}_0) \qquad W = O(z^3)$

<u>Thm</u> Let $W, W' = O(z^3)$

$$R\mathrm{End}_{D_{sg}(W^{-1}(0))}(\mathcal{O}_0) \xrightarrow{\sim} R\mathrm{End}_{D_{sg}(W'^{-1}(0))}(\mathcal{O}_0)$$

$$\begin{array}{ccc} H^{\cdot}(B_W) & \xrightarrow{\sim} & H^{\cdot}(B_{W'}). \\ \wr\| & & \wr\| \\ \Lambda(V) & \xrightarrow{id} & \Lambda(V) \end{array}$$

Then W' can be obtained from W by a formal change of variables $\quad z_i \longmapsto z_i + O(z^2)$.

$\mathcal{O}_0 \in D_{sg}(W^{-1}(0)) \qquad$ find a Matrix factorization

$$S \xrightarrow{a} S \xrightarrow{b} b \quad \langle a, b \rangle$$

$y_1, y_2, \cdots, y_n \quad \sum_i y_i z_i = W. \quad y_i = O(z^2)$

$E = \bigotimes_{i=1}^{n} \langle y_i, z_i \rangle \in MF(W)$.

$(E, \delta) \qquad \mathrm{cok}(\delta^1) \cong \mathcal{O}_0$ in $D_{sg}(W^{-1}(0))$

$B_W = \mathrm{End}_{MF(W)}(E)$.

$E = \Lambda(V)$. $\dim V = n$ $\quad y = \sum z_k \frac{\partial}{\partial z_k}$ $\quad \gamma = \sum_k g_k \, dz_k$

$\iota_y + \gamma \wedge$

$B_W = \Lambda(V) \otimes \Lambda(V)$

$w \otimes \theta$

$w \wedge \iota_\theta(\cdot)$

differential $\partial : B_W \longrightarrow B_W$

$\partial(w \otimes \theta) = \iota_y(w) \otimes \theta$

$(\tilde{\delta} - \delta)(w \otimes \theta) = \sum_k y_k w \otimes \iota_{dz_k}\theta$

$\iota : \Lambda(V) \longrightarrow B_W$

$\iota(\theta) = 1 \otimes \theta$

$p : B_W \longrightarrow \Lambda(V)$

$\Lambda(V) \underset{p}{\overset{\iota}{\rightleftarrows}} (B_W, \delta)$

$h : B_W \longrightarrow B_W$

$\tilde{\iota} : \Lambda(V) \longrightarrow (B_W, \tilde{\delta})$. quasi-isomorphism.

$\Lambda(V) \cong H^\bullet(B_W)$

$H^\bullet(B_W) \cong \Lambda(V)$

A – any algebra

$m_n : A^{\otimes n} \longrightarrow A$. $n \geq 3$. $\deg(m_n) = 2 - n$.

$m_2(ab) = ab$

$m_1 = 0$

dg alg $\mathfrak{g}$'s d component

$\mathfrak{g}^d = \prod_{\substack{(1) = d+1 \\ i \geq d+2}} \mathrm{Hom}^i(A^{\otimes i}, A)$ $\quad \Rightarrow \quad \mathfrak{g} = \bigoplus_d \mathfrak{g}^d$

$(m_n, n \geq 3) \longleftrightarrow \alpha \in \mathfrak{g}^1$

$d\alpha + \frac{1}{2}[\alpha, \alpha] = 0$

$$\mathfrak{g}^0 \longrightarrow \mathrm{Vect}(\mathfrak{g}^1)$$
$$\gamma \longmapsto (\alpha \mapsto -\partial\gamma + [\gamma, \alpha])$$

one gets group action $G^0 = \exp(\mathfrak{g}^0)$

$$G^0 \cdot MC(\mathfrak{g}^1)$$

$$\mathcal{MC}(\mathfrak{g}) = MC(\mathfrak{g})$$

Lemma. $\mathfrak{g}_1, \mathfrak{g}_2$ pro-nilpotent. DGLA.

$$\Phi : \mathfrak{g}_1 \to \mathfrak{g}_2 \text{ - filtered qis.}$$

then $\varphi_* : \mathcal{MC}(\mathfrak{g}_1) \longrightarrow \mathcal{MC}(\mathfrak{g}_2)$

$h = \prod \mathrm{Hom}^j(\Lambda(V)^{\otimes j}, \Lambda(V))^*$ $\qquad$ h - DGLA. pro nilp.

$$W \in S \rightsquigarrow \alpha \in MC(h)$$
$$W' \in S \rightsquigarrow \alpha' \in MC(h).$$
$$\bar{\alpha} = \bar{\alpha}' = \mathcal{MC}(h)$$

Manifold, M.

$\theta \in \Gamma(\Lambda^2 TM)$ section of tangent. bundle.

Possion $[\theta, \theta] = 0$.

$$T^d_{poly}(M) = \Gamma(\Lambda^{d+1} TM) - \mathfrak{g}la$$

$$f_1 * f_2 = f_1 f_2 + D_1(f_1, f_2) h + D_2(f_1, f_2) h^2 + \cdots$$

$D_{poly}(M)$ — DGLA

$$\varphi : T_{poly}(M) \xrightarrow{L_\infty} D_{poly}(M)$$

$\varphi_*(\theta)$ — deformation quant.

$W \quad W'$

$\downarrow \quad \swarrow$

$\alpha \, \alpha'$

$\cap$

$\mathcal{MC}(h)$

$\bar{\alpha} = \bar{\alpha}'$

$\leftarrow$ ring of power series.

$$\mathfrak{g} = k[x_1, \cdots, x_n] \otimes \Lambda\langle \xi_1, \cdots \xi_n \rangle \otimes k[\hbar]$$

$f \in k[x_1, \cdots, x_n]$, $\{f, f\} = 0$

φ

$\varphi : h \longrightarrow g$

$\varphi_*(\alpha) \sim W$

$g^\circ \subseteq g$, $G^\circ = \exp(g^\circ)$

$G^\circ \supseteq \mathrm{Aut}(k[\![x_1, \dots, x_n]\!])$

$x_i \mapsto x_i + O(x^2)$.