A Direct Approach to Searching with Lies *

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Abstract The searching problem when one of M integers is unknown and has to be found with n questions under the assumption that e lies are allowed in the answers is considered. It is shown that any searching strategy can be described using rooted regular binary trees introduced in a special way. This description leads to the conclusion that the problem has a solution only if a more difficult problem, when a certain number of integers such that no lies are allowed in the answers are added to the searching domain, can be solved with the same number of questions. As a result, we rediscover some known results with simpler arguments, prove a new upper bound on the number of questions that differs from the known lower bound by $3e$, and present a sub-optimal algorithm for constructing the searching strategy.

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Search for a pattern, Trees, Coverings

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1 Introduction

Let us consider the following game (see Fig.1). Suppose that there is a board containing two rows and $e+1$ columns numbered by integers $0, \ldots, e$ and a player who is given $c_0 + \ldots + c_e$ coins. The player distributes c_j coins among the rows of the j-th column, $j = 0, \ldots, e$. Then another player, called the devil, puts a pointer either on the first or the second row. After that all coins of the selected row are shifted by 1 position to the left; the coins of the leftmost column are removed from the board. These actions represent the first move of the game. As a result of the move, the player receives new numbers of coins in each column, and the participants make the second move. The game is finished when the board is either empty, or contains only 1 coin, no matter in which column. The goal of the player is to finish the game as fast as possible, while the devil tries to play as long as possible. Some positions that are attained in the game when the player is initially given 4 coins in column 0 and 1 coin in column 1 are shown in Fig.1, where the pointer drawn in the middle of the board means that the devil does not distinguish between the variants. Note that if the player distributes the 4 coins of column 0 into equal halves at the first move, then the devil can make the game consist of 4 moves. However, if these coins are distributed in the proportion $1:3$ and the proportion in column 1 is $1:0$, then the game is finished after at most 3 moves whatever the devi does.

The game described above relates to the problem of the search for an unknown integer m chosen from the set $\{1, \ldots, M\}$, where M is given. Suppose that this integer is chosen by the "devil". We ask him a question, i.e., present some subset $\mathcal{M}$ of the set $\{1, \ldots, M\}$ and receive the answer "yes" if $m \in \mathcal{M}$ and "no" if $m \notin \mathcal{M}$. After that we form the next question based on this and previous answers. Suppose that the devil is allowed to lie by answering "yes" if $m \notin \mathcal{M}$ and "no" if $m \in \mathcal{M}$, but the number of lies is not greater than a given number e. Furthermore, he is allowed to change m during the searching procedure in such a way that the integer he currently has in mind satisfies the rules of the game, i.e., he lied for this integer not more than e times. The positions presented in Fig.1 correspond to the case when $M = 5$, $e = 1$, the question at time instant -1 was : is $m \in \{1\}$, and the answer was "yes". As a result, we count 1 and 0 lies if $m \in \{2, 3, 4, 5\}$ and $m = 1$, respectively, and generate the vector $\mathbf{c}_0 = (4, 1)$. The distribution of coins at time instant 0 in