Universität Bielefeld

Fakultät für Mathematik

SFB 701

Infinite Dimensional Analysis and Representation Theory

10 – 14 December 2012

Department of Mathematics
Bielefeld University
Location: Main Building, V2–210/216

This workshop is part of the conference program of the DFG-funded CRC 701
Spectral Structures and Topological Methods in Mathematics
at Bielefeld University.

Organizers: Michael Baake, Friedrich Götze, Yuri Kondratiev, Holger Kösters, Henning Krause, Claus Michael Ringel, Michael Röckner

http://www.math.uni-bielefeld.de/sfb701/2012_IDART

Programme

Monday, 10 December 2012

8:50 – 9:00 *Opening*

9:00 – 10:00 **Anatoly Vershik** (Steklov Institute of Mathematics, St. Petersburg)
Classification of functions via randomization, random matrices and dynamics of metrics I

10:05 – 11:05 **Andrei Zelevinsky** (Northeastern University, Boston)
Quantum cluster algebras and their triangular bases I

Coffee Break

11:30 – 12:30 **Marek Bozejko** (University of Wrocław)
Generalized Gaussian processes with application to noncommutative functional analysis I

Lunch Break

14:00 – 15:00 **Anatoly Vershik** (Steklov Institute of Mathematics, St. Petersburg)
Classification of functions via randomization, random matrices and dynamics of metrics II

Coffee Break

15:25 – 16:25 **Dmitri Finkelshtein** (Institute of Mathematics, Kiev)
Harmonic analysis on configuration spaces and related convolution algebras

16:30 – 17:30 **Xiao-Wu Chen** (University of Science and Technology of China, Hefei)
Singularity categories, Leavitt path algebras and shift spaces

Programme

Tuesday, 11 December 2012

9:00 – 10:00	**Craig A. Tracy** (UC Davis) Bethe ansatz methods in stochastic integrable models
10:05 – 11:05	**Anatoly Vershik** (Steklov Institute of Mathematics, St. Petersburg) Classification of functions via randomization, random matrices and dynamics of metrics III
	Coffee Break
11:30 – 12:30	**Grigori Olshanski** (Russian Academy of Sciences, Moscow) Determinantal measures and Markov dynamics I
	Lunch Break
14:00 – 15:00	**Philippe Bougerol** (Université Paris VI) Random walks and semisimple groups I
	Coffee Break
15:25 – 16:25	**Christian Stump** (Leibniz Universität Hannover) The history of noncrossing partitions
16:30 – 17:30	**Kai-Uwe Bux** (Bielefeld University) Noncrossing partitions and classifying spaces for braid groups (after T. Brady)

Programme

Wednesday, 12 December 2012

9:00 – 10:00	**Andrei Zelevinsky** (Northeastern University, Boston) Quantum cluster algebras and their triangular bases II
	Coffee Break
10:30 – 11:30	**Marek Bozejko** (University of Wrocław) Generalized Gaussian processes with application to noncommutative functional analysis II
	Lunch
13:00	*Excursion* (Bauernhausmuseumscafé)

Programme

Thursday, 13 December 2012

9:00 – 10:00	**Christoph Schweigert** (Universität Hamburg) Invariants of mapping class groups for logarithmic vertex algebras I
10:05 – 11:05	**Sergej Kuksin** (École Polytechnique, Palaiseau) Analysis of the KdV equation and its perturbations
	Coffee Break
11:30 – 12:30	**Philippe Bougerol** (Université Paris VI) Random walks and semisimple groups II
	Lunch Break
14:00 – 15:00	**Christoph Schweigert** (Universität Hamburg) Invariants of mapping class groups for logarithmic vertex algebras II
	Coffee Break
15:25 – 16:25	**Uwe Franz** (Université de Franche-Comté, Besançon) The quantum symmetry group of a Hadamard matrix
16:30 – 17:30	**Martin Venker** (Bielefeld University) Repulsive particle systems and random matrices
19:00	*Conference Dinner* (Restaurant Bültmannshof)

Programme

Friday, 14 December 2012

9:00 – 10:00	**Grigori Olshanski** (Russian Academy of Sciences, Moscow) Determinantal measures and Markov dynamics II
10:05 – 11:05	**Andrei Zelevinsky** (Northeastern University, Boston) Quantum cluster algebras and their triangular bases III
	Coffee Break
11:30 – 12:30	**Marek Bozejko** (University of Wrocław) Generalized Gaussian processes with application to noncommutative functional analysis III
	Lunch

Abstracts

Philippe Bougerol (Université Paris VI)

Random walks and semisimple groups

A random walk on a group G is the product $S_n = g_1 g_2 \cdots g_n$ of random variables (g_k) with values in G, independent and with the same distribution.

In the first talk I will illustrate the fact that simple transformations of random walks in the (commutative) weight lattice of a semi simple complex group G can describe the Littelmann theory of finite dimensional representations of these groups. In the second talk I will show that some random walks on G itself give an interpretation of the geometric crystal of Berenstein and Kazhdan. By tropicalization, one recovers Littelmann theory. This is a survey of the following works:

[1] Biane, Ph., Bougerol, Ph, O'Connell, N. Littelmann paths and Brownian paths. Duke Math. J. 130 (2005), no. 1, 127-167.

[2] Biane, Ph., Bougerol, Ph, O'Connell, N. Continuous crystal and Duistermaat-Heckman measure for Coxeter groups. Adv. Maths. 221 (2009) 1522-1583.

[3] Chhaibi Reda, Modèle de Littelmann pour cristaux géométriques, fonctions de Whittaker sur des groupes de Lie et mouvement brownien (Paris 6 thesis)

Marek Bozejko (University of Wrocław)

Generalized Gaussian processes with application to noncommutative functional analysis

In my talks I will consider the following subjects: (a) q-Gaussian processes, theta functions of Jacobi, second quantization and connections with new von Neumann algebras. Also we will consider connections with the free probability (case $q = 0$) and we give characterisation of (b) Free Levy processes. Also we present the Meixner laws and the role of the classical Nevanlinna-Pick theorem about analytic functions on the upper half-plane. Next we will give the role of (c) Hecke-Yang-Baxter operators for the construction of new models of noncommutative probability like monotone probability, Boolean probability and others. Connections with some new models of operator spaces and noncommutative Khinchine inequality will be also done.

References:

[1] M. Bozejko, On Lambda(p) sets with minimal constant in discrete noncommutative groups, Proc. Amer. Math. Soc. 51 (1975), 407-412.

[2] M. Bozejko and R. Speicher, An example of a generalized Brownian motion, Comm. Math. Phys. 137 (1991), 519-531.

[3] M. Bozejko and R. Speicher, Completely positive maps on Coxeter groups, deformed commutation relations and operator spaces, Math. Annalen, 300, 97-120(1994).

[4] M. Bozejko and R. Speicher, Interpolation between bosonic and fermionic relations given by generalized Brownian motion, Math. Zeitsch., 222, 135-160.

[5] M. Bozejko., B. Kummerer and R. Speicher, q-Gaussian Processes: Non-commutative and Classical Aspects, Comm. Math. Phys. 185, 129-154 (1997).

[6] M. Bozejko, Ultracontractivity and strong Sobolev inequality for q-Ornstein-Uhlenbeck semigroup $(-1 < q < 1)$; Infinite Dimensional Analysis, Quantum Probability and Related Topics 2 (1999), 203-220.

[7] M. Bozejko and M. Guta, Functors of white noise associated to characters of the infinite symmetric group, Comm. Math. Phys. 229 (2002), 209-227.

[8] M. Bozejko and W. Bryc, On a class of free Levy laws related to a regression problem, J. Funct. Anal. 236 (2006), 59-77.

[9] M. Bozejko and E. Lytvynov, Meixner class of non-commutative generalized stochastic processes with freely independent values. I. Characterization, Comm. Math. Phys. 292 (2009), 99-129.

[10] M. Anshelevich, S. Belinschi, M. Bozejko and F. Lehner, Free infinite divisibility for q-Gaussians, Math. Res. Lett.17 (2010), 909-920.

[11] S. Belinschi, M. Bozejko, F. Lehner and R. Speicher, The normal distribution is free infinitely divisible, Adv. in Math. 226 (2011), 3677-3698.

[12] M. Bozejko, Deformed Fock spaces, Hecke operators and monotone Fock space of Muraki, Demonstratio. Math. 45 (2012), 399-413.

Kai-Uwe Bux (Bielefeld University)

Noncrossing partitions and classifying spaces for braid groups (after T. Brady)

Let G be a group. A classifying space for G is a CW-complex with fundamental group G and contractible universal cover. This cover is a contractible CW-complex with a free action of G. Conversely, any such contractible free G-complex gives rise to a classifying space. Classifying spaces are unique up to homotopy equivalence and their homotopy type is an important topological invariant of the group. Searching within this homotopy type for particularly nice representatives is an important part of geometric group theory.

In this commissioned talk, I shall consider the case where G is the braid group B_n on n strands. Tom Brady has constructed a particularly nice classifying space for this group making essential use of noncrossing partitions. I shall explain his construction.

Xiao-Wu Chen (University of Science and Technology of China, Hefei)

Singularity categories, Leavitt path algebras and shift spaces

To a finite quiver (= a finite oriented graph), one associates three objects of differential types: the singularity category of the corresponding finite-dimensional algebra with radical square zero, the Leavitt path algebra and the shift space. We will explain that these three objects are closely related to each other.

Dmitri Finkelshtein (Institute of Mathematics, Kiev)

Harmonic analysis on configuration spaces and related convolution algebras

Harmonic analysis on the space of locally finite subsets (configurations) of underlying spaces is connected to the proper combinatorial-type convolutions on the space of functions of finite subsets. We consider two types of convolutions for such functions and study their properties and relations. The corresponding calculus w.r.t. one type of the convolutions is considered and the connections with the convolution of the corresponding states (measures) are established. We study also derivative operations w.r.t. this convolution and apply them for describing of the evolution for cummulants of the measures.

Uwe Franz (Université de Franche-Comté, Besançon)

The Quantum Symmetry Group of a Hadamard Matrix

To each complex Hadamard matrix one can associate a unique "quantum symmetry group" (or quantum permutation group, i.e. a subgroup of the free permutation compact quantum group S_N^+). Many questions about the subfactors and planar algebras associated to a Hadamard matrix have an equivalent formulations in terms of its quantum symmetry group. In my talk I will present a probabilistic approach to characterising this quantum symmetry group and study several examples in small dimension. For 4×4 Hadamard matrices one obtains twists of the dihedral groups D_N. Based on joint work with Teodor Banica, Franz Lehner, and Adam Skalski.

Sergej Kuksin (École Polytechnique, Palaiseau)

Analysis of the KdV equation and its perturbations

It is known since 1960's that "the KdV equation is integrable", but exact analytical meaning of this assertion became clear only recently. And even now it is not clear "how integrable is KdV". Right answer to this question is important for physics, where often not KdV but its perturbations are used. To study perturbed KdV exact properties of the transformation which integrates KdV and of the KdV hamiltonian are crucial. In my talk I will discuss the corresponding results and open problems.

References:

[1] T. Kappeler, J. Poschel, *KdV & KAM*, Springer 2003.

[2] S. Kuksin, Gal. Perelman, *Vey theorem in infinite dimensions and its application to KdV*, DCDS-A 27 (2010), 1-24.

[3] E. Korotyaev, S. Kuksin, *KdV Hamiltonian as function of actions*, arXiv 2011.

Grigori Olshanski (Russian Academy of Sciences, Moscow)

Determinantal measures and Markov dynamics

My two talks are based on joint work with Sergey Pirogov, [7]. We construct a model of Markov dynamics for a one-dimensional lattice gas system with infinitely many interacting particles. The model possesses a stationary distribution, which is a determinantal measure meaning that its correlation functions are given by principal minors of a kernel $K(x, y)$ on the lattice. (About determinantal measures, see e.g. Borodin [1], Soshnikov [8].)

The kernel $K(x, y)$ first appeared in Borodin–Olshanski [3]; it has the same general structure

$$\frac{A(x)B(y) - B(x)A(y)}{x - y}$$

as the so-called integrable kernels from Random Matrix Theory. We call $K(x, y)$ the Gamma kernel, because A and B are expressed through Euler's Γ-function.

In the equilibrium regime, reproducing the stationary distribution, the dynamical correlation functions of our dynamics also have determinantal form. That is, they are given by appropriate minors of an "extended" kernel $K(x, s;\, y, t)$ with two space-time arguments, (x, s) and (y, t). The structure of this "extended Gamma kernel" is similar to that of the extended kernels from Random Matrix Theory (see Tracy–Widom [9]).

The present work is related to previous works [4], [5], [6] by Borodin and myself, devoted to dynamical models for some continuous gas systems. Both continuous and lattice models in question originated from some problems of representation theory (see Borodin–Olshanski [2]). However, the methods used in the continuous case and in the lattice case are quite different.

In the lattice case, our approach is based on a close connection between determinantal measures and their "noncommutative" analogs — quasifree states on the algebra CAR of canonical anticommutation relations.

References:

[1] Alexei Borodin, *Determinantal point processes*. In: The Oxford Handbook of Random Matrix Theory (Gernot Akemann, Jinho Baik and Philippe Di Francesco, editors), Oxford University Press, 2011, pp. 231–249; arXiv:0911.1153.

[2] Alexei Borodin and Grigori Olshanski, *Distributions on partitions, point processes and the hypergeometric kernel*. Communications in Mathematical Physics **211** (2000), 335–358; arXiv: math.RT/9904010.

[3] Alexei Borodin and Grigori Olshanski, *Random partitions and the Gamma kernel*. Advances in Mathematics **194** (2005), 141–202; arXiv: math-ph/0305043.

[4] Alexei Borodin and Grigori Olshanski, *Markov processes on the path space of the Gelfand-Tsetlin graph and on its boundary*. Journal of Functional Analysis **263** (2012), 248–303; arXiv:1009.2029.

[5] Grigori Olshanski, *Laguerre and Meixner symmetric functions, and infinite-dimensional diffusion processes*. Zapiski Nauchnyh Seminarov POMI **378** (2010), 81-110; reproduced in Journal of Mathematical Sciences (New York) **174** (2011), no. 1, 41–57; arXiv:1009.2037.

[6] Grigori Olshanski, *Laguerre and Meixner orthogonal bases in the algebra of symmetric functions*. International Mathematics Research Notices **2012** (2012), 3615–3679; arXiv:1103.5848.

[7] Grigori Olshanski and Sergey Pirogov, *Paper in preparation.*

[8] Alexander Soshnikov, *Determinantal random point fields*. Uspekhi Matematicheskih Nauk **55** (2000), No. 5, 107–160 (Russian); English translation: Russian Mathematical Surveys **55** (2000), 923–975; arXiv:math/0002099.

[9] Craig A. Tracy and Harold Widom, *Differential Equations for Dyson Processes*. Communications in Mathematical Physics **252** (2004), 7–41; arXiv:math/0302033.

Christoph Schweigert (Universität Hamburg)

Invariants of mapping class groups for logarithmic vertex algebras

Vertex algebras are infinite-dimensional algebraic structures describing conserved quantities in two-dimensional quantum field theories. In certain cases, their representation category can be related to the category of left modules over a finite-dimensional ribbon Hopf algebra. Surprisingly, this Kazhdan-Lusztig duality works particularly well in a case when the representation category is not semi simple.

After having reviewed this work of Feigin, Semikhatov et al., we will explain how to construct from these categories invariants of mapping class groups of Riemann surfaces that are candidates for physical partition functions (joint work with J. Fuchs and C. Stigner).

Christian Stump (Leibniz Universität Hannover)

The history of noncrossing partitions

I will start with noncrossing set partitions as studied by Germain Kreweras in the 1970. Then, I will discuss the combinatorics and geometry of noncrossing partitions as elements in finite (real and complex) reflection groups studied by various authors in the past 15 years. Finally, I will discuss their connections to the geometry of root systems and to cluster algebras and categories.

Craig A. Tracy (UC Davis)

Bethe Ansatz Methods in Stochastic Integrable Models

The asymmetric simple exclusion process (ASEP) is not of determinantal class, but it is integrable in the sense that Bethe Ansatz methods lead to an explicit formula for the transition probability for N-particle ASEP. We sketch the proof of this fact. Two variations of ASEP are multispecies ASEP (first, second, etc. class particles) and ASEP on the semi-infinite lattice $\{0, 1, 2, \ldots\}$. In both cases the standard Bethe Ansatz has to be modified in order to compute the transition probability of the N-particle system. We explain these modifications and indicate how the proof of the main result is changed. This is joint work with Harold Widom.

Martin Venker (Bielefeld University)

Repulsive Particle Systems and Random Matrices

We consider particle systems on the real line in which particles experience repulsion of power beta when getting close. This is the same as for the beta-ensembles from random matrix theory. For distances larger than zero, the interaction in the new ensembles is allowed to differ from those present for random eigenvalues. We show that the local bulk correlations of the beta-ensembles, universal in random matrix theory, also appear in the new ensembles. The result extends the bulk universality classes of random matrix theory and may provide an explanation for the appearance of random matrix statistics in a number of situations which do not have an obvious interpretation in terms of random matrices. Partly joint with Friedrich Götze.

References:

[1] F. Götze, M. Venker: Local Universality of Repulsive Particle Systems and Random Matrices, arxiv.org/abs/1205.0671

[2] M. Venker: Particle Systems with Repulsion Exponent beta and Random Matrices, arxiv.org/abs/1209.3178

Anatoly Vershik (St. Petersburg Department of Steklov Institute of Mathematics)

Classification of functions via randomization, random matrices and dynamics of metrics

1. The problem about classification of functions of several variables. Matrix distribution of the functions, properties of the random matrices.

2. A function of two variables as a random field, analog of Kolmogorov theorem; canonical model of function. Theorem about reconstruction of the function with given matrix distribution.

3. Invariant measures and the link between classification and Aldous theorem. Examples, Problems.

Andrei Zelevinsky (Northeastern University, Boston)

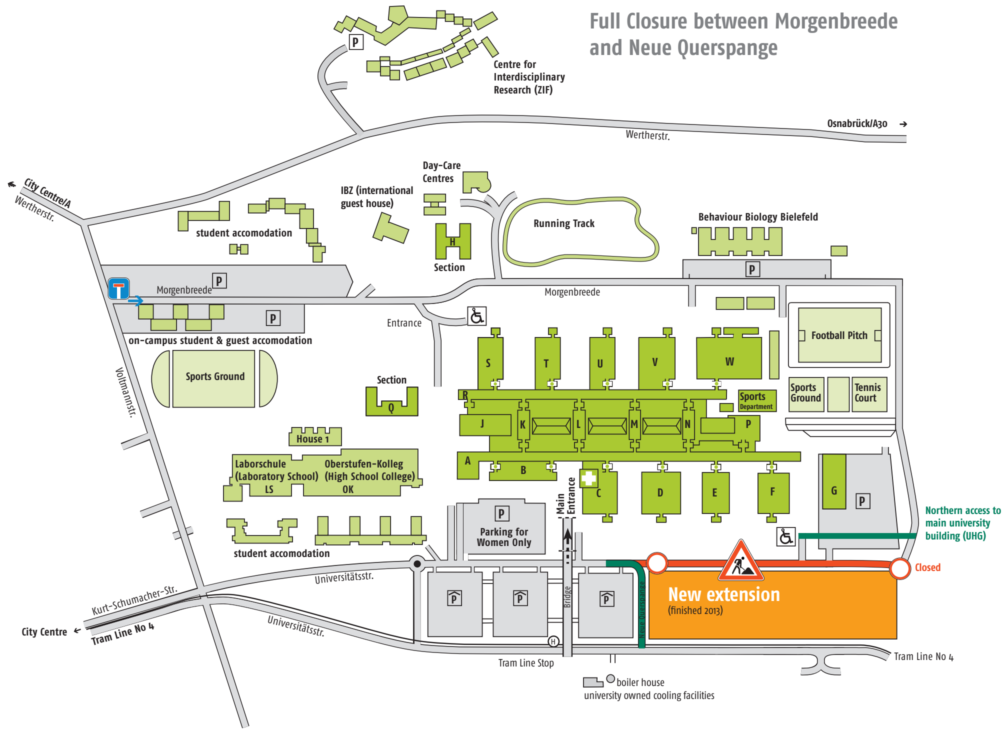
Quantum cluster algebras and their triangular bases

Cluster algebras were discovered by Sergey Fomin and myself about a decade ago. One of our main motivations was to understand an algebraic framework behind Lusztig's canonical basis for quantum groups. With this goal in mind, in 2005 Arkady Berenstein and myself have introduced quantum cluster algebras. Very recently (in the Summer 2012), we have also found a version of Lusztig's original approach that is applicable to a wide class of quantum cluster algebras. This leads to an elementary construction of a class of bases we call triangular.

The rough plan of my three lectures is as follows. Lecture 1 will give an introduction to cluster algebras focusing on the facts (mostly from the joint paper "Cluster algebras III" with A.Berenstein and S.Fomin) most relevant for the quantization procedure. Lecture 2 will discuss quantum cluster algebras, and Lecture 3 will be devoted to triangular bases.

Getting to Bielefeld University

Take tram line 4 with direction “Lohmannshof” until tram line stop “Universität”. The Department of Mathematics resides in part V of the main university building (green in the map below) at the levels 2 – 6.



Campus Map