A MATHEMATICAL MISTAKE IN N. BOURBAKI'S "GENERAL TOPOLOGY"

My favorite books in General Topology are the books of N. Bourbaki. There are a lot of reasons for that. Among them I strongly believe that especially the part of Exercises is an endless source of deep results and a continuous inspiration for further research. But even in the legendary "General Topology" of N. Bourbaki there is at least one mistake! Precisely in the Exercise 13 of Ch. X, p. 323, part (d) in [1] it is said that if E is a uniformly equicontinuous family of homeomorphisms of a locally compact uniform space X then $K(E) := \{x \in X \ : \ Ex \text{ is relatively compact}\}$ is a closed subset of X. This is not true if E is not a subset of a uniformly equicontinuous *group* of homeomorphisms of X as we can easily see by the following counterexample.

Counterexample. Let

$$X = \bigcup_{k=1}^{\infty} \{(x,y) \,:\, x = \frac{1}{k},\, y \geq 0\} \cup \{(x,y) \,:\, x = 0,\, y > 0\}$$

be endowed with the Euclidean metric. Consider the family $E = \{f_n\}$ of selfmaps of X defined by $f_n(x,y) = (x, \frac{y}{n})$. The family E consists of uniformly equicontinuous homeomorphisms of X and $K(E) = \bigcup_{k=1}^{\infty} \{(x,y) \,:\, x = \frac{1}{k},\, y \geq 0\}$ as can be easily checked. Hence the set $K(E)$ is not closed in X.

References

[1] Bourbaki N., *Elements of Mathematics. General Topology*, Parts I and II, Hermann, Paris, 1966.