Journal of Pure and Applied Algebra **104** *(1995) 235-241*

Induction for Finite Groups Revisited

ANTHONY BAK
Department of Mathematics
University of Bielefeld
33501 Bielefeld, Germany

Abstract *Let G be a finite group and $I(G)$ a nonempty family of subgroups of G, which is closed under conjugation and taking subgroups of its members. Let F be a Green ring functor on G. Let Ω be the Burnside ring functor on G and let Ω_F denote the image of the canonical natural transformation $\Omega \longrightarrow F$. Suppose that for each subgroup $H \subset G$, every $\mathbb{Z}$-torsion element of $\Omega_F(H)$ is nilpotent. Then the following are equivalent: (1) F is $I(G)$-hypercomputable. (2) Ω_F is $I(G)$-hypercomputable. (3) The restriction homomorphism $Res_{I(G)}^{\,G}(\mathbb{Q} \otimes F) : \mathbb{Q} \otimes_{\mathbb{Z}} F(G) \longrightarrow \prod_{H \in I(G)} \mathbb{Q} \otimes_{\mathbb{Z}} F(H)$ is injective. (4) the induction homomorphism $Ind_{I(G)}^{\,G}(\mathbb{Q} \otimes F) : \coprod_{H \in I(G)} \mathbb{Q} \otimes_{\mathbb{Z}} F(H) \longrightarrow \mathbb{Q} \otimes_{\mathbb{Z}} F(G)$ is surjective.*

1. Introduction

We recall the notions of Mackey functor, Green ring functor, Green module, and hypercomputability for Mackey functors, as well as the universal properties of the Burnside ring functor. A detailed exposition of these notions is included in [BM, § 7].

Let G be a finite group. Let $Sub(G)$ denote the category whose objects are all subgroups $H \subset G$ and whose morphism are all symbols $i_{H,g,K} : H \longrightarrow K$ where H and K are subgroups of G and $g \in G$ such that $gHg^{-1} \subset K$. Composition is defined by $i_{K,g',K'} i_{H,g,K} = i_{H,g'g,K'}$. Each morphism $i_{H,g,K}$ has a natural realization as a group homomorphism $H \longrightarrow K, h \longmapsto ghg^{-1}$, but distinct morphisms can have the same realization. A **Mackey functor** *on G is a bifunctor $M : Sub(G) \longrightarrow$ ((abelian groups)) [G, (1.3) axioms G1 and G2] satisfying the Mackey subgroup property [G, (1.3) axioms G3 and G4]. If M is a Mackey functor on G, one often lets $Ind_{g,H}^{\;K} M = M_\star(i_{H,g,K})$ and $Res_{g,H}^{\;K} M = M^\star(i_{H,g,K})$ and sets $Ind_H^K = Ind_{1,H}^{\;K}$ and $Res_H^K = Res_{1,H}^{\;K}$. The map $Ind_{g,H}^{\;K} M$ (resp. $Res_{g,H}^{\;K} M$) is called an* **induction map** *(resp.*

restriction map). *If $I(G)$ is a family of subgroups of G, let $Ind_{I(G)}^G M$ denote the canonical map $\coprod_{H\in I(G)} M(H) \longrightarrow M(G)$, $\coprod_{H\in I(G)} x_H \longmapsto \sum_{H\in I(G)} Ind_H^G M(x_H)$, and let $Res_{I(G)}^G M$ denote the canonical map $M(G) \longrightarrow \prod_{H\in I(G)} M(H), x \longmapsto \prod_{H\in I(G)} Res_H^G M(x)$.*

Let $I(G)$ be a nonempty family of subgroups of G closed under conjugation and taking subgroups of its members. A Mackey functor M on G is called $I(G)$**-computable** *if the canonical maps $\underset{I(G)}{colim}\, M_\star \longrightarrow M(G)$ and $M(G) \longrightarrow \lim_{I(G)} M^\star$ induced by induction and restriction, respectively, are bijective. Let S be a set consisting of the natural number 1 and some natural primes p, possibly none, such that p divides $|G| :=$ order of G. Let $I(G)^S = \{H \subset G \mid \ \exists N \trianglelefteq H, N \in I(G), \mid H/N \mid \ = \text{power of } s, s \in S\}$. Let S' denote the multiplicative subset of $\mathbb{Z}$ generated by 1 and all natural primes p such that $p \notin S$ and p divides $|G|$. A Mackey functor M on G is called* $I(G)$**-hypercomputable** *if for all sets S as above, the functor $S'^{-1}M := S'^{-1}\mathbb{Z} \otimes_{\mathbb{Z}} M$ is $I(G)^S$-computable.*

Let ((associative rings with identity))^w denote the category whose objects are all associative rings with identity, whose morphisms are all identity preserving ring homomorphisms between associative rings with identity, and whose weak morphisms (accounting for the superscript w above) are all group homomorphisms between associative rings with identity. A **Green ring functor** *on G is a bifunctor $F : Sub(G) \longrightarrow$ ((associative rings with identity))^w such that for any morphism $i_{H,g,K} \in Sub(G), F^\star(i_{H,g,K})$ is a morphism in ((associative rings with identity))^w and $F_\star(i_{H,g,K})$ is a weak morphism in ((associative rings with identity))^w, F satisfies the Mackey subgroup property (in particular, F is a Mackey functor), and F satisfies the Frobenius reciprocity law [G, (1.3) axiom G5].*

A Mackey functor M on G is called a **Green module** *over a Green ring functor F on G if M is a Frobenius module [L, Chap. III §1] over F.*

Let

$$\Omega : Sub(G) \longrightarrow \ \text{((commutative associative rings with identity))}^w$$

denote the Burnside ring functor. It is well known, cf. [t.D, §1], that Ω is a Green ring functor, that every Mackey functor on G is canonically a Green module over Ω, and that for any Green ring functor F on G, there is *a canonical natural transformation* $\Omega \longrightarrow F$ *of Green ring functors such that for each subgroup* $H \subset G$, *image* $(\Omega(H) \longrightarrow F(H)) \subset$ *center* $(F(H))$. *Let*

$$\Omega_F := \text{ image } (\Omega \longrightarrow F).$$

The following theorem is dual to A. Dress' induction theorems [D, (1.2) and (1.7)]. A formulation of Dress' results, in the language of the present article is given in [B, (12.13)].

THEOREM 1.1. *Let* G *be a finite group and* $I(G)$ *a nonempty family of subgroups of* G, *which is closed under conjugation and taking subgroups of its members. Let* F *be a Green ring functor on* G *such that for each subgroup* $H \subset G$, *every* $\mathbb{Z}$*-torsion element of* $F(H)$ *is nilpotent. Then the restriction homomorphism* $Res_{I(G)}^{G}(\mathbb{Q}\otimes F) : \mathbb{Q}\otimes_{\mathbb{Z}} F(G) \longrightarrow \prod_{H\in I(G)} \mathbb{Q}\otimes_{\mathbb{Z}} F(H)$ *is injective* $\Longleftrightarrow$ F *is* $I(G)$*-hypercomputable.*

The theorem above allows one to prove a result which could be called the

FUNDAMENTAL THEOREM 1.2. *Let* G *be a finite group and* $I(G)$ *a nonempty family of subgroups of* G, *which is closed under conjugation and taking subgroups of its members. Let* F *be a Green ring functor on* G *such that for each subgroup* $H \subset G$, *every* $\mathbb{Z}$*-torsion element of* $\Omega_F(H)$ *is nilpotent. Then the following are equivalent:*

(1) F *is* $I(G)$*-hypercomputable.*
(2) Ω_F *is* $I(G)$*-hypercomputable.*
(3) The homomorphism $Res_{I(G)}^{G}(\mathbb{Q} \otimes F) : \mathbb{Q} \otimes_{\mathbb{Z}} F(G) \longrightarrow \prod_{H\in I(G)} \mathbb{Q} \otimes_{\mathbb{Z}} F(H)$ *is injective.*
(4) The homomorphism $Ind_{I(G)}^{G}(\mathbb{Q} \otimes F) : \coprod_{H\in I(G)} \mathbb{Q} \otimes_{\mathbb{Z}} F(H) \longrightarrow \mathbb{Q} \otimes_{\mathbb{Z}} F(G)$ *is surjective.*

PROOF *(1)* $\Rightarrow$ *(2). (1) implies that the restriction homomorphism* $Res_{I(G)}^{G}(\mathbb{Q}\otimes F) : \mathbb{Q}\otimes_{\mathbb{Z}} F(G) \longrightarrow \prod_{H\in I(G)} \mathbb{Q}\otimes_{\mathbb{Z}} F(H)$ *is injective. It follows that the restriction homomorphism* $Res_{I(G)}^{G}(\mathbb{Q}\otimes\Omega_F) : \mathbb{Q}\otimes_{\mathbb{Z}}\Omega_F(G) \longrightarrow \prod_{H\in I(H)} \mathbb{Q}\otimes_{\mathbb{Z}}\Omega_F(H)$ *is injective. One checks easily that every subbifunctor of a Green ring functor is a Green ring functor. Hence,* Ω_F *is a Green ring functor and by Theorem (1.1),* Ω_F *is* $I(G)$*-hypercomputable.*

(2) ⇒ (3). Since F is a Green module over Ω_F and Ω_F is $I(G)$-hypercomputable, it follows from Dress' result [D, (1.2)] (cf. [B, (12.13)(a)]) that F is $I(G)$-computable. In particular, the restriction homomorphism $Res_{I(G)}^{G}(\mathbb{Q} \otimes F) : \mathbb{Q} \otimes_{\mathbb{Z}} F(G) \longrightarrow \prod_{H \in I(G)} \mathbb{Q} \otimes_{\mathbb{Z}} F(H)$ is injective.

(3) ⇒ (4). (3) implies, as in the proof of (1) ⇒ (2), that the restriction homomorphism $Res_{I(G)}^{G}(\mathbb{Q} \otimes_{\mathbb{Z}} \Omega_F) : \mathbb{Q} \otimes_{\mathbb{Z}} \Omega_F(G) \longrightarrow \prod_{H \in I(G)} \mathbb{Q} \otimes_{\mathbb{Z}} \Omega_F(H)$ is injective. Thus, by Theorem (1.1), Ω_F is $I(G)$-hypercomputable. Hence, as in the proof of (2) ⇒ (3), F is $I(G)$-hypercomputable. In particular, the induction homomorphism $Ind_{I(G)}^{G}(\mathbb{Q} \otimes F) : \coprod_{H \in I(G)} \mathbb{Q} \otimes_{\mathbb{Z}} F(H) \longrightarrow \mathbb{Q} \otimes_{\mathbb{Z}} F(G)$ is surjective.

(4) ⇒ (1). This follows from Dress induction [D, (1.2) and (1.7)]. Q.E.D.

The following corollaries of the fundamental theorem have applications in K-theory [BM, (12.22) and (12.23)], surgery theory [BM, (13.20)], and transformation groups [BM, §14].

Corollary 1.3. *Let G be a finite group and $I(G)$ a nonempty family of subgroups of G, which is closed under conjugation and taking subgroups of its members. Let F be a Green ring functor on G such that for each subgroup $H \subset G$, every $\mathbb{Z}$-torsion element of $\Omega_F(H)$ is nilpotent. Then if F is $I(G)$-hypercomputable, every submackey functor $F'' \subset F$ and every quotient Mackey functor F' of F is $I(G)$-hypercomputable.*

Proof *The important observation to make is that F' and F'' are Green modules over Ω_F (but not necessarily over F). By the Fundamental Theorem (1.2), Ω_F is $I(G)$-hypercomputable. Thus, by [D, (1.2)], F' and F'' are $I(G)$-hypercomputable.*

Corollary 1.4. *Let G be a finite group and $I(G)$ a nonempty family of subgroups of G, which is closed under conjugation and taking subgroups of its members. Let $F_0, F_1, \ldots, F_m$ be a sequence of Green ring functors on G such that for each $i > 0$, one of the following holds: (1) There is a natural transformation $F_{i-1} \longrightarrow F_i$ of Green ring functors. (2) The canonical action of Ω on F_i factors through $\Omega_{F_{i-1}}$ and for each subgroup $H \subset G$, every $\mathbb{Z}$-torsion element of $\Omega_{F_{i-1}}(H)$ is nilpotent. (3) There is an exact sequence $F_i'' \longrightarrow F_i \xrightarrow{\gamma} F_{i-1}$ of natural transformations of Mackey functors (in particular, γ is not necessarily a natural transformation of Green ring functors)* *such that F_i'' is $I(G)$-hypercomputable and for each subgroup $H \subset G$, every $\mathbb{Z}$-torsion element of $\Omega_{F_{i-1}}(H)$ is nilpotent. (4) There is an exact sequence $F_i'' \longrightarrow F_i \xrightarrow{\gamma} F_{i-1}$ of Mackey functors such that F_i'' is $I(G)$-hypercomputable and γ is surjective. Then F_0 is $I(G)$-hypercomputable $\Longleftrightarrow$ all the F_i's are $I(G)$-hypercomputable.*

PROOF. *The assertion $\Leftarrow$) is trivial. The assertion $\Rightarrow$) is proved by induction on m. The case $m = 0$ is trivial. Assume that $m > 0$ and that the result is true for $m-1$. Thus, F_{m-1} is $I(G)$-hypercomputable. Let $F = F_m$ and $F' = F_{m-1}$.*

Suppose there is a natural transformation $F' \longrightarrow F$ as in (1). Then F is a Green module over F'. Since F' is $I(G)$-hypercomputable, it follows from [D, (1.2)] that F is $I(G)$-hypercomputable.

Suppose (2) holds. Then F is a Green module over $\Omega_{F'}$ and by (1.2), $\Omega_{F'}$ is $I(G)$-hypercomputable. It follows now from [D, (1.2)] that F is $I(G)$-hypercomputable.

Let $F'' \longrightarrow F \xrightarrow{\gamma} F'$ be an exact sequence as in (3). Let S and S' be as in the definition of $I(G)$-hypercomputable. Consider the commutative diagram of short exact sequences.

$$\begin{array}{ccccc}
\coprod_{H\in I(G)^S} S'^{-1}F''(H) & \longrightarrow & \coprod_{H\in I(G)^S} S'^{-1}F(H) & \twoheadrightarrow & \coprod_{H\in I(G)^S} S'^{-1}\ \mathit{image}\ \gamma(H) \\
Ind_{I(G)^S}^{G}(S'^{-1}F'')\downarrow & & Ind_{I(G)^S}^{G}(S'^{-1}F)\downarrow & & \downarrow Ind_{I(G)^S}^{G}(S'^{-1}\ \mathit{image}\ \gamma) \\
S'^{-1}F''(G) & \longrightarrow & S'^{-1}F(G) & \twoheadrightarrow & S'^{-1}\ \mathit{image}\ \gamma(G).
\end{array}$$

By [D, (1.2)], it suffices to show that the induction map $Ind_{I(G)^S}^{G}(S'^{-1}F)$ is surjective. By assumption, the induction map $Ind_{I(G)^S}^{G}(S'^{-1}F'')$ is surjective and by (1.3), the induction map $Ind_{I(G)^S}^{G}(S'^{-1}image\gamma)$ is surjective. Thus, by the 5-Lemma, the induction map $Ind_{I(G)^S}^{G}(S'^{-1}F)$ is surjective.

Let $F'' \longrightarrow F \xrightarrow{\gamma} F'$ be an exact sequence as in (4). Then one gets a commutative diagram as above and deduces as above that F is $I(G)$-hypercomputable. Q.E.D.

The rest of this article is devoted to the proof of Theorem (1.1).

2. Proof of Theorem (1.1)

Throughout this section, G denotes a finite group and F a Green ring functor on G.

DEFINITION 2.1. *Let $I(G)$ and $R(G)$ be nonempty families of subgroups of G, which are closed under conjugation and taking subgroups of their members. $I(G)$ is called* **relatively F-prime to** $R(G)$ *if the 2-sided ideal image $(Ind_{I(G)}^{G}F)$ is relatively prime to the 2-sided ideal $Ker(Res_{R(G)}^{G}F)$, i.e., image $(Ind_{I(G)}^{G}F) + Ker(Res_{R(G)}^{G}F) = F(G)$.*

LEMMA 2.2. *Let F' be a Green ring functor on G which is a Green module over F. If $I(G)$ is relatively F-prime to $R(G)$ then $I(G)$ is relatively F'-prime to $R(G)$.*

PROOF. *Let $x \in$ image $(Ind_{I(G)}^{G}F)$ and $y \in Ker\ (Res_{R(G)}^{G}F)$ such that $x+y = 1_{F(G)}$. Clearly, $1_{F'(G)} = 1_{F(G)}1_{F'(G)} = x1_{F'(G)} + y1_{F'(G)}$. By Frobenius reciprocity, $x1_{F'(G)} \in$ image $(Ind_{I(G)}^{G}F')$. Furthermore, $y1_{F'(G)} \in Ker(Res_{R(G)}^{G}F')$ because for $H \in R(G), Res_H^G F'(y1_{F'(G)}) = (Res_H^G F(y))(Res_H^G F'\ (1_{F'(G)})) = 0$. Q.E.D.*

PROPOSITION 2.3. *Let F' be a Green ring functor on G. Let $I(G)$ and $R(G)$ be nonempty families of subgroups of G, which are closed under conjugation and taking subgroups of their members, such that $I(G)$ is relatively F-prime to $R(G)$. If F' is a Green module over F and $Ker(Res_{R(G)}^{G}F') = 0$ then every Green module on G over F' is $I(G)$-computable.*

PROOF. *By Dress' theorem [D, (1.2)] (cf. [B, (12.13) (a)]), it suffices to show that the induction map $Ind_{I(G)}^{G}F'$ is surjective. But, this is a trivial consequence of Lemma (2.2) and the assumption that $Ker(Res_{R(G)}^{G}F') = 0$. Q.E.D.*

Let $\mathcal{S}(G)$ denote the set of all subgroups of G. If $\mathcal{I} \subset \mathcal{S}(G)$ is closed under conjugation, let $Conj(\mathcal{I})$ denote the set of conjugation classes (H) of elements $H \in \mathcal{I}$. Give $Conj(\mathcal{S}(G))$ the partial ordering defined by the rule $(H) \leqq (K) \Leftrightarrow H$ is conjugate to a subgroup of K.

DEFINITION 2.4. *A linear ordering $(H_1), (H_2), \ldots, (H_r)$ of the elements of $Conj(\mathcal{S}(G))$ is called* **good** *if $(H_i) \leqq (H_j) \Longrightarrow i \leqq j$. A nonempty, conjugate closed subset $I(G) \subset \mathcal{S}(G)$ is called* **initial** *if there is a good ordering $(H_1), (H_2), \ldots, (H_r)$ of $Conj(\mathcal{S}(G))$ such that for some k, $Conj(I(G)) = \{(H_1), (H_2), \ldots, (H_k)\}$.*

The next lemma characterizes initial families of subgroups of G.

LEMMA 2.5. *Let $I(G)$ be a nonempty, conjugate closed family of subgroups of G. Then $I(G)$ is an initial family $\Longleftrightarrow$ $I(G)$ is closed under taking subgroups of its members.*

PROOF *The proof is an elementary exercise.*

KEY LEMMA 2.6. *Let $\Omega : Sub(G) \longrightarrow$ ((commutative associative rings with identity*
$))^w$ denote the Burnside ring (bi)functor. Let $I(G)$ be a nonempty family of subgroups of G, which is closed under conjugation and taking subgroups of its members. Then $I(G)$ is relatively $\mathbb{Q} \otimes_{\mathbb{Z}} \Omega$-prime to $I(G)$.

PROOF. *By Lemma (2.5), there is a good ordering $(H_1), (H_2), \ldots, (H_r)$ of $Conj(\mathbb{S}(G))$ such that $\{(H_1), (H_2), \ldots, (H_k)\} = Conj(I(G))$, for some k. For $(H) \in Conj(\mathbb{S}(G))$ and a finite G-set X, define $\phi_{(H)}(X) = |X^H| :=$ number of elements in X which are left fixed by every element in H. It is easy to check, cf. [t.D, §1], that $\phi_{(H)}$ defines a ring homomorphism $\phi_{(H)} : \Omega(G) \longrightarrow \mathbb{Z}$. Let $e_i (1 \leq i \leq r)$ denote the idempotent of $\prod^r \mathbb{Z}$ whose i'th coordinate is 1 and other coordinates are 0. Define $\phi : \Omega(G) \longrightarrow \prod^r \mathbb{Z}, X \longmapsto \prod_{i=1}^r \phi_{(H_i)}(X) e_i$. Give the Burnside ring $\Omega(G)$ the ordered $\mathbb{Z}$-basis $[G/H_1], [G/H_2], \ldots, [G/H_r]$ where $[G/H_i]$ denotes the isomorphism class of the G-set G/H_i of left cosets of H_i in G. Give $\prod^r \mathbb{Z}$ the ordered $\mathbb{Z}$-basis $e_1, e_2, \ldots, e_r$. One deduces easily from the paragraph preceeding [t. D, (1.2.3)] that ϕ is represented by an upper triangular matrix $\mathbb{M}(\phi)$ whose i'th diagonal element is $|N_G(H_i)/H_i|$ where $N_G(H_i)$ is the normalizer of H_i in G. In particular, the diagonal entries of $\mathbb{M}(\phi)$ are nonzero natural numbers. Thus, ϕ is injective and $\mathbb{Q} \otimes \phi : \mathbb{Q} \otimes_{\mathbb{Z}} \Omega(G) \longrightarrow \prod^r \mathbb{Q}$ is a ring isomorphism.*

It is well known that the Burnside ring functor is a Green ring functor on G. Thus, the bifunctor $\mathbb{Q} \otimes_{\mathbb{Z}} \Omega : Sub(G) \longrightarrow$ ((commutative associative rings with identity$))^w$, $H \longmapsto \mathbb{Q} \otimes_{\mathbb{Z}} \Omega(H)$, is also a Green ring functor. Let $x \in \mathbb{Q} \otimes_{\mathbb{Z}} \Omega(G)$. It follows from the definition of restriction for the Burnside ring functor that $Res_H^G(\mathbb{Q} \otimes_{\mathbb{Z}} \Omega)(x) = 1_{\mathbb{Q} \otimes_{\mathbb{Z}} \Omega(H)} \Longleftrightarrow (\mathbb{Q} \otimes \phi_{(K)})(x) = 1$ for all subgroups K of H. Thus, $I(G)$ is relatively $\mathbb{Q} \otimes_{\mathbb{Z}} \Omega$-prime to $I(G) \Longleftrightarrow$(by definition) there are elements $x \in$ image $(Ind_{I(G)}^G(\mathbb{Q} \otimes_{\mathbb{Z}} \Omega))$ and $y \in Ker(Res_{I(G)}^G(\mathbb{Q} \otimes_{\mathbb{Z}} \Omega))$ such that $x + y = 1_{\mathbb{Q} \otimes_{\mathbb{Z}} \Omega(G)} \Longleftrightarrow$ there is an element $x \in$ image $(Ind_{I(G)}^G(\mathbb{Q} \otimes_{\mathbb{Z}} \Omega))$ such that $(Res_H^G(\mathbb{Q} \otimes_{\mathbb{Z}} \Omega))(x) = 1_{\mathbb{Q} \otimes_{\mathbb{Z}} \Omega(H)}$ for all $H \in I(G) \Longleftrightarrow$ there is an element $x \in$ image $(Ind_{I(G)}^G(\mathbb{Q} \otimes_{\mathbb{Z}} \Omega))$ such that $(\mathbb{Q} \otimes \phi_{(H)})(x) = 1$ for all $H \in I(G)$. Since $\mathbb{Q} \otimes \phi$ is bijective, there *is a unique element* $x \in \mathbb{Q} \otimes_{\mathbb{Z}} \Omega(G)$ *such that* $(\mathbb{Q} \otimes \phi_{(H)})(x) = 1$ *for all* $H \in I(G)$ *and* $(\mathbb{Q} \otimes \phi_{(H)})(x) = 0$ *for all* $H \notin I(G)$. *Since* $\mathbb{M}(\mathbb{Q} \otimes \phi) = \mathbb{M}(\phi)$ *is a triangular matrix with nonzero diagonal entries and since* $Conj(I(G)) = \{(H_1), (H_2), \ldots, (H_k)\}$, *the element* x *above has the form* $\sum_{i=1}^{k} a_i [G/H_i]$ *for suitable rational numbers* $a_i \in \mathbb{Q}$. *Thus,* $x \in$ *image* $(Ind_{I(G)}^{G}(\mathbb{Q} \otimes_{\mathbb{Z}} \Omega))$. *Q.E.D.*

PROOF OF THEOREM 1.1. *By Dress induction [D, (1.2) and (1.7)] (a formulation in the language of the present article is given in [B, (12.13)]), it suffices to show that the induction map* $Ind_{I(G)}^{G}(\mathbb{Q} \otimes_{\mathbb{Z}} F)$ *is surjective. But, this follows immediately from (2.6), (2.3), and the fact that* $\mathbb{Q} \otimes_{\mathbb{Z}} F$ *is a Green module over* $\mathbb{Q} \otimes_{\mathbb{Z}} \Omega$, *because any Mackey functor, for example* F, *is a Green module over* Ω. *Q.E.D.*

References

B. *A. Bak, K-Theory of Forms, Annals Math. Studies* **98**, *Princeton Univ. Press, Princeton (1981)*

BM. *A. Bak and M. Morimoto, Equivariant Surgery on Compact Manifolds with Half Dimensional Singular Sets, preprint*

D. *A. Dress, Induction and Structure theorems for orthogonal representations of finite groups, Ann. Math.* **102** *(1975), 291 – 325*

t.D. *T. tom Dieck, Transformation Groups and Representation Theory, Lec. Notes Math.* **766** *(1979), Springer-Verlag*

G. *J.A. Green, Axiomatic Representation Theory for Finite Groups, J. Pure Appl. Alg.* **1** *(1971), 41 – 77*

L. *T.-Y. Lam, Induction theorems for Grothendieck groups and Whitehead groups of finite groups, Ann. Ec. Norm. Sup. (4) 1 (1968), 91 – 148*