

# Fredholm Properties and $L^p$ -Spectra of Localized Rotating Waves in Parabolic Systems

Denny Otten<sup>1</sup>  
Department of Mathematics  
Bielefeld University  
33501 Bielefeld  
Germany

**Abstract.** Rotating waves are special solutions of reaction-diffusion systems which rotate at constant velocity while maintaining their shape. Nonlinear stability results for such waves are usually based on spatial behavior of the wave profile and on spectral properties of the linearization. The linearization, obtained by linearizing the co-rotating frame at the wave profile, turns out to be an additive variable coefficient perturbation of a complex-valued Ornstein-Uhlenbeck operator.

In this talk we first present a short review about previous results on exponential decay of rotating waves. We then investigate Fredholm properties of the linearization, derive the Fredholm alternative and show under suitable assumptions that eigenfunctions and their adjoints decay exponentially in space. We then provide different techniques to derive certain subsets of the spectrum of the linearization. The main idea is to reduce the eigenvalue problem of the linearized operator to a finite-dimensional one. For this purpose, we first derive the dispersion set, which is affected by the far-field behavior of the wave, and show that it belongs to the essential  $L^p$ -spectrum. We then derive the symmetry set, which is induced by the underlying group symmetries, and show that it belongs to the point  $L^p$ -spectrum. From Fredholm properties we deduce exponential decay of the associated eigenfunctions and their adjoints. Finally, we present numerical results for spinning solitons that appear in the cubic-quintic complex Ginzburg-Landau equation.

---

<sup>1</sup>e-mail: [dotten@math.uni-bielefeld.de](mailto:dotten@math.uni-bielefeld.de), phone: +49 (0)521 106 4784,  
fax: +49 (0)521 106 6498, homepage: <http://www.math.uni-bielefeld.de/~dotten/>.  
supported by CRC 701 'Spectral Structures and Topological Methods in Mathematics', Bielefeld University