Exponential decay of two-dimensional rotating waves

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November 15, 2011

In this talk we show exponential decay in space and time for solution kernels of complex-valued parabolic systems in several dimensions with negative absolute term. The convection terms of these systems have unbounded coefficients and they are of rotational type.

A model equation in $\mathbb{R}^2$ is given by

$$U_t = A\triangle U + cD_\phi U + B(x)U,\ x \in \mathbb{R}^2$$

where $U(x,t) \in \mathbb{C}^N$, $0 \neq c \in \mathbb{R}$, $A \in \mathbb{C}^{N,N}$ is positive definite and $B(x) \in \mathbb{C}^{N,N}$ converges to some negative definite matrix as $|x| \to \infty$. By D_ϕ we denote the angular derivative which in Cartesian coordinates reads $D_\phi := -x_2 D_1 + x_1 D_2$.

For the constant coefficient operator obtained for $|x| \to \infty$ we derive a representation for the solution kernel from the Feynman-Kac formula. This is used to show exponential decay in time and space for solutions of the variable coefficient inhomogeneous equation. Moreover, this also leads to exponential decay for the stationary equation.

This study is motivated by the stability problem for rotating waves in several variables.