REPRESENTATION THEORY
EXERCISES 3

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1. Let $\mathcal{A}$ be an abelian category and $\mathcal{C}$ a Serre subcategory of $\mathcal{A}$. Prove the following:

(a) If $\mathcal{A}$ is noetherian, then so are $\mathcal{C}$ and $\mathcal{A}/\mathcal{C}$.

(b) If $\mathcal{A}$ is artinian, then so are $\mathcal{C}$ and $\mathcal{A}/\mathcal{C}$.

(c) Neither the converse of (a) nor the converse of (b) is true.

Hint for (c): Consider the matrix ring $\left(\begin{smallmatrix} \mathbb{R} & \mathbb{R} \\ 0 & \mathbb{Q} \end{smallmatrix}\right)$, which is left noetherian and left artinian but neither right noetherian nor right artinian.

2. (a) Explain why the condition (Tr4) in the definition of a triangulated category is known as the octahedral axiom.

(b) Consult the literature (e.g. Neeman's book *Triangulated Categories* or Hubery's *Notes on the Octahedral Axiom*) and compare different formulations of (Tr4). Pick the one you consider the most elegant.

3. (a) Prove the equivalence of the following statements for a ring homomorphism $\phi\colon A \to B$:

(i) ϕ is an epimorphism.

(ii) The map $B \otimes_A B \to B$ induced by ϕ is an isomorphism.

(iii) Restriction of scalars $\operatorname{Mod} B \to \operatorname{Mod} A$ via ϕ is fully faithful.

(b) Compute the universal localization of $A = \left(\begin{smallmatrix} \mathbb{Z} & \mathbb{Q} \\ 0 & \mathbb{Z} \end{smallmatrix}\right)$ with respect to $\Sigma = \{e_2 A \xrightarrow{e_{12}\cdot} e_1 A\}$.

To be handed in via email by May 11, 2020, 2 p.m.