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Nearness Spaces – a Constructive Approach

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ABSTRACT

We introduce an axiomatic constructive development of the theory of **nearness** and **apartness** for a point and a set as a framework for constructive topology. Using first-order logic, we start with a set X equipped with three primitive relations—inequality, apartness and nearness—which satisfy a number of natural axioms. We also study a second-order theory with only two primitive relations —inequality and apartness—in which nearness is *defined* by

$$\text{near}(x, A) \Longleftrightarrow \forall B(\text{apart}(x, B) \Longrightarrow \exists y \in A - B).$$

In this theory the number of axioms is substantially lower than in its first-order counterpart.

We then study the relationship between the nearness topology induced by a nearness relation and the topological nearness induced by a given topology. In the second-order theory we show that the topological nearness induced by the nearness topology can be identified with the original nearness. Finally, we obtain some elementary results on the continuity of mappings between nearness spaces.

References

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