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Fuzzy functions: A fuzzy extension of the category *SET*

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ABSTRACT

In research works where fuzzy sets are involved, in particular, in fuzzy topology, fuzzy algebra, mostly certain usual functions are taken as morphisms: they can be mappings between corresponding sets, or between the fuzzy powersets of these sets, etc. On the other hand there are only few papers where attempts *to fuzzify the concept of a function* are undertaken. The aim of our work is to present a possible approach to this problem. Namely, a certain class of L-relations (i.e. mappings $f : X \times Y \to L$) is distinguished which can be viewed as (L-)*fuzzy functions* from a set X to a set Y. As the result we obtain a fuzzy category [3] LFST (see Definition below).

Let $L = (L, \leq, \wedge, \vee, *)$ be an infinitely distributive GL-monoid (cf. [1]), and let $\longmapsto$ be the corresponding residuation (i.e. $\alpha * \beta \leq \gamma \Longleftrightarrow \alpha \leq \beta \longmapsto \gamma$).

Definition By $L - FSET(L)$ we denote the category whose objects are triples (X, E, A) where (X, E) is an L-valued set and A is its strict extensional L-subset [1] and whose morphisms, called *(potential) fuzzy functions*, from (X, E_X, A) to (Y, E_Y, B) are L-mappings $F : X \times Y \to L$ such that $\forall x, x' \in X; \forall y, y' \in Y$ the following four properties hold:
(0ff) $F(x, y) \leq E_X(x, x) \wedge E_Y(y, y)$;
(1ff) $\sup_x A(x) * \big(E_X(x, x) \longmapsto F(x, y)\big) \leq B(y)$;
(2ff) $F(x, y) * \big(E_Y(y, y) \longmapsto E_Y(y, y')\big) \leq F(x, y')$
(3ff) $E_X(x, x') * \big(E_X(x, x) \longmapsto F(x, y)\big) \leq F(x', y)$;
(4ff) $F(x, y) * \big(E_X(x, x) \longmapsto F(x, y')\big) \leq E_Y(y, y')$.
Given a fuzzy function $F : (X, E_X, A) \to (Y, E_Y, B)$ let $\mu(F) = \inf_x \sup_y F(x, y)$.
For two fuzzy functions $F : (X, E_X, A) \to (Y, E_Y, B)$ and $G : (Y, E_Y, B) \to (Z, E_Z, C)$ the *composition* $G \circ F : (X, E_X, A) \to (Z, E_Z, C)$ is defined by the formula
$(G \circ F)(x, z) = \bigvee_{y \in Y} \Big(\big(F(x, y) * \big(E_Y(y, y) \to G(y, z)\big)\Big)$. A direct verification shows that $G \circ F$ is a fuzzy function and that $\mu(G \circ F) \geq \mu(G) * \mu(F)$. Further, given an L-valued set (X, E) let $\omega(X, E) := \mu(E) = \inf_x E(x, x)$. Thus a *fuzzy category* [3] $(L - FSET(L), \omega, \mu)$ is obtained.

We study *images* and *preimages* of L-sets under fuzzy functions; introduce properties of *injectivity* and *surjectivity* for them, etc. Lastly some fuzzy categories related to topology and algebra and having certain fuzzy functions in the role of morphisms are itroduced and their properties are studied.

References

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