

## An Open Problem in Abstract Algebraic Logic

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### ABSTRACT

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#### *Is every selfextensional logic fully selfextensional?*

Neither a proof, nor a counterexample, has been found up to now.

A *logic* is here a sentential logic in the most general sense (i.e., in TARSKI's sense): A pair  $\mathcal{S} = \langle \mathcal{L}, \vdash_{\mathcal{S}} \rangle$  where  $\mathcal{L}$  is an algebraic language, and  $\vdash_{\mathcal{S}} \subseteq P(\mathbf{Fm}_{\mathcal{L}}) \times \mathbf{Fm}_{\mathcal{L}}$  is the *consequence relation* of  $\mathcal{S}$ ;  $\mathbf{Fm}_{\mathcal{L}}$  is the absolutely free  $\mathcal{L}$ -algebra with denumerably many variables.

A logic  $\mathcal{S}$  is *selfextensional* (WÓJCICKI, 1979) when the relation of *interderivability*  $\dashv\vdash_{\mathcal{S}}$  is a congruence of the formula algebra  $\mathbf{Fm}_{\mathcal{L}}$ , where  $\phi \dashv\vdash_{\mathcal{S}} \psi \iff \{\phi\} \vdash_{\mathcal{S}} \psi$  and  $\{\psi\} \vdash_{\mathcal{S}} \phi$ .

A logic is *fully selfextensional* (FONT and JANSANA, 1996) when all its *full models* have the similar property, that is, the property that the relation of interderivability, or *Frege relation*, of the model is a congruence of the algebra reduct of the model. A *full model* of a sentential logic is a special kind of *generalized matrix* (WÓJCICKI, 1969; also called *abstract logics* by BLOOM, BROWN and SUSZKO, 1973). It was introduced and studied in [3].

This problem is of a typical kind in *abstract algebraic logic*, called *transfer problems*: Given an abstract or metalogical property that can be predicated both of a logic and of its models, to find out whether all its full models have it whenever the logic has it. In this case, the interest of the problem comes from: (1) The good behaviour that has been empirically observed of many fully selfextensional logics, specially concerning their being definable by Gentzen systems whose models, in a natural sense, are precisely the full models of the logic. (2) The partial results that have already been obtained: The answer is YES if additionally the logic has a *conjunction*, or if it satisfies the ordinary *deduction theorem* for some *implication* connective.

#### Main references for Abstract Algebraic Logic

- [1] BLOK, W and PIGOZZI, D. *Algebraizable Logics*. Memoirs of the A.M.S., nr. 396. Providence, 1989. 78 pp.
- [2] CZELAKOWSKI, J. *Protoalgebraic Logics*. Trends in Logic, Studia Logica Library, to appear. Kluwer, Dordrecht, 2000.
- [3] FONT, J.M. and JANSANA, R. *A General Algebraic Semantics for Sentential Logics*. Lecture Notes in Logic, vol. 7. Springer-Verlag, Berlin and Heidelberg, 1996. 135 pp.

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