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On Groups in which normality is a transitive relation.

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ABSTRACT

The aim of this paper is to give two new characterizations of finite solvable groups in which every subnormal subgroups is normal in G (T-groups) and to characterize the finite groups all of those subgroups are either normal or self-normalizing. We also investigate properties of $\mathcal{H}$-subgroups, where a subgroup H of G is called an $\mathcal{H}$-subgroup of G if

$$N_G(H) \cap H^g \leq H \quad \forall\ g \in G.$$

The set of all $\mathcal{H}$-subgroups of G will be denoted by $\mathcal{H}(G)$. If G is a finite group, the following statements are equivalent:

- G is soluble T-group
- $\mathcal{H}(G) = \mathcal{L}(G)$, where $\mathcal{L}(G)$ is the lattice of subgroups of G
- Every p-subgroup of G is an $\mathcal{H}$-subgroup.

Consequently, using the previous result, if G is a finitely generated group, the following statement are equivalent:

- G is a supersoluble T-group
- G is soluble and $\mathcal{H}(G) = \mathcal{L}(G)$.

As a corollary we prove a characterization of the finite groups G in which each subgroup is either normal or selfnormalizing ($\mathcal{NSN} - group$). Let G be a finite group. Then G is an $\mathcal{NSN}$-group iff either G is a Dedekind group or G satisfies the following conditions:

- G contains an abelian normal subgroup of prime index p in G
- If $P \in Syl_P(G)$ is cyclic and self normalizing in G
- If $P = \langle x \rangle$ then there exists an integer n such that $w^x = W^n$ for all $w \in H$.

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