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Subgroup-permutability and affine planes

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ABSTRACT

Let G be a finite group. Its subgroup-permutability graph $\Gamma(G)$ is a graph whose vertices are the non normal subgroups of G and two vertices are adjacent if and only if the two corresponding subgroups permute. There exist groups G such that $\Gamma(G)$ is isomorphic to the incidence graph $\Gamma(A)$ of a finite affine plane A, i.e.to a graph whose vertices are the points and lines of A, and two vertices are adjacent if and only if they are two incident lines, or a point and a line such that the point is on the line.

The aim of this paper is to classify these groups.Indeed, we prove the following result:

$\Gamma(G)$ is isomorphic to the incidence graph $\Gamma(A)$ of an affine plane of order n if and only if:a) There exist two primes p, q such that $n = p$, $p \equiv 1 \pmod{q}$, $|G| = p^2q^k$, $k \geq 1$.

b) $G = [P]Q$ where P is elementary abelian of order p^2, Q is cyclic of order q^k, and $|Z(G)| = q^{k-1}$.

c) Q acts on P as a power automorphism group of order p.

As a consequence we obtain that the affine plane A is desarguesian, and there is no group G such that $\Gamma(G)$ is isomorphic to the incidence graph of a projective plane.

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