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On the metric theory of diophantine approximation in the field of p-adic numbers

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ABSTRACT

We will deal with Diophantine approximation of p-adic numbers. The metric theorem of Khintchine [1] is generalized. K. Mahler (1934) first investigated similar problem in $\mathbf{Z}_p$.

Let $p \geq 2$ be a prime number, $\mathbf{Q}_p$ be the field of p-adic numbers with the Haar measure, $\mathbf{Z}_p$ be the ring of p-adic integers, $|\dots|_p$ be the p-adic valuation. Let $n \in \mathbf{N}$, $n \geq 2$, $\Psi(n) : \mathbf{N} \to \mathbf{R}^+$ be monotonic and $S_1(\Psi)$ be the set of $x \in \mathbf{Q}_p$ such that the inequality $|F(x)|_p < h^{-n}\Psi(h)$ holds for infinitely many $F \in \mathbf{Z}[x]$, $\deg F = n$, where $h = h(F)$ is the height of F.

Theorem 1. (E.Kovalevskaya, 1998) *If* $\sum_{h=1}^{\infty} \Psi(h) < \infty$ *then the set* $S_1(\Psi)$ *has zero Haar measure.*

The following theorem is about Diophantine approximation of general planar curves in $\mathbf{Z}_p \times \mathbf{Z}_p$. Let $f : \mathbf{Z}_p \to \mathbf{Z}_p$, $f(x) = \sum_{n=1}^{\infty} c_n(x-c)^n$ be a normal function such that $f''(x) \neq 0$ almost everywhere on $\mathbf{Z}_p$. Let $G(x) = b_0 + b_1 x + b_2 f(x)$, where $b_i \in \mathbf{Z}$ $(i = 0, 1, 2)$, $H = \max |b_i| \neq 0$, and let $S_2(\Psi)$ be the set of $x \in \mathbf{Z}_p$ such that the inequality $|G(x)|_p < H^{-2}\Psi(H)$ holds for infinitely many G.

Theorem 2. (V.Beresnevich and E.Kovalevskaya, 1999) *The set* $S_2(\Psi)$ *has full or zero Haar measure according as the series* $\sum_{H=1}^{\infty} \Psi(H)$ *diverges or converges.*

We develop the methods of V. Sprindzuk (1963) and Yu. Melnichuk (1981).

Reference

[**1**] *A.Khintchine,* Einige Sätze über Kettenbrüche mit Anwendungen auf die Theorie der Diophantischen Approximationen, Math. Ann. **92** (1924), 115-125.

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