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On the restricted Waring problem for polynomial cubes

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ABSTRACT

The Waring problem for polynomial cubes over a finite field F of characteristic p, consists of finding the minimal integer $m \geq 0$, such that every sum of cubes in $F[t]$ is a sum of m cubes. It is known that for F distinct from $F_2, F_4, F_7, F_{13}, F_{16}$, each polynomial in $F[t]$ is a sum of three cubes of polynomials. (See [4]).

If a polynomial $P \in F[t]$ is a sum of n cubes of polynomials in $F[t]$, such that each cube A^3 appearing in the decomposition has degree $< \deg(P) + 3$, we say that P is a restricted sum of n cubes.

The restricted Waring problem for polynomial cubes consists of finding the minimal integer $m \geq 0$, such that each sum of cubes S in $F[t]$ is a restricted sum of m cubes.

We denote this integer m by $rw(p, F)$.

In 1993 M.Car and J. Cherly proved that for all q even every polynomial $P \in F_q[t]$ of sufficiently high degree is a restricted sum of at most 11 cubes, by using the circle method. (See [1]).

Recently, (See [2]), we improved this result using elementary methods to $rw(2, F_q) \leq 9$ for q even, $q \notin \{2, 4, 16\}$, and $rw(2, F_{16}) \leq 10$.

In common work with M.Car, (See [3]) we improved the above results and extended them to odd q as follows:

One has $rw(p, F_q) \leq 7$ for $q \notin \{2, 4, 16, 7, 13\}$, $rw(7, F_7) \leq 9$, $rw(13, F_{13}) \leq 9$, $rw(2, F_{16}) \leq 8$.

We are working with M.Car on a new idea for reducing the above bounds.

References

[1] M. Car and J. Cherly, *Sommes de cubes dans l'anneau $F_{2^n}[X]$*, Acta Arith. 65, Number 3, 227–241, 1993.

[2] L. Gallardo, *On the restricted Waring problem over $F_{2^n}[t]$*. To appear in Acta Arith. 2000.

[3] M. Car and L. Gallardo, *Sums of cubes in $F_q[T]$*. Preprint.

[4] L. N. Vaserstein, *Sums of cubes in Polynomial Rings,* Math. of Comp. Vol. 56, Number 193, January 1991, pp. 349–357.

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