

Non-abelian tensor product modulo q of Lie algebras and applications

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ABSTRACT

The notion of non-abelian tensor (exterior) product modulo q , $M \otimes^q N (M \wedge^q N)$, where q is a positive integer and M, N are crossed P -modules in the context of Lie algebras, is introduced and some properties are established. The general aim introducing this notion is to describe the universal q -central relative extension of a Lie epimorphism, analogously to Conduché-Rodríguez-Fernández's result in the group case.

Compatibility of non-abelian tensor (exterior) product modulo q of crossed modules with the direct limit of crossed modules is established. The definition and some examples of crossed squares of Lie algebras are given. Using a slightly generalized version of Whitehead's universal quadratic functor the relation between the Lie exterior product modulo q and Lie tensor product modulo q is studied. The relation between Ellis' non-abelian tensor product and the non-abelian tensor product modulo q of two Lie algebras with compatible actions on each other is given.

We introduce the definition of a q -central relative extension of a Lie epimorphism, which is the mod q version of Kassel-Loday's notion of relative extension and give our main result: for a Lie epimorphism $\alpha : P \rightarrow Q$ there exists a universal q -central relative extension of α if and only if $\text{Ker}\alpha$ coincides with the submodule of P generated by the elements $[n, p]$ and qn for $n \in \text{Ker}\alpha$, $p \in P$ and this extension is given by

$$0 \rightarrow V \rightarrow (\text{Ker}\alpha) \wedge^q P \rightarrow P \rightarrow Q \rightarrow 0.$$

Keywords: *Lie algebra, tensor product modulo q , exterior product modulo q , crossed module, universal q -central relative extension*

Mathematics Subject Classification: *18G50*

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