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On semiperfect rings satisfying the Engel condition

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ABSTRACT

Let R be an associative ring and R° its adjoint group. The multiplication in R° is given by $a \circ b = a + b + ab$ for all a and b in R. If R is a ring with identity $1 \neq 0$ then $U(R)$ will denote the group of units of R and moreover $R^\circ \cong U(R)$. Let $[x, y] = x \circ y \circ x' \circ y'$ is a commutator of x and y in R°, where $x \circ x' = x' \circ x = 0 = y \circ y' = y' \circ y$. For any positive integer n we can define the n-th commutator $[x, {}_n y]$ by the rule $[x, {}_{n+1} y] = [[x, {}_n y], y]$. A group G is called an Engel group if every element x of G is an Engel one (i.e. for each g in G there is an integer $n = n(x.g) \geq 0$ such that $[x, {}_n g] = 1$).

Let us recall that a ring R is semilocal if the quotient $R/J(R)$ is right Artinian, and semilocal ring R is semiperfect if all idempotents of $R/J(R)$ can be lifted modulo $J(R)$ to idempotents of R.

Proposition 1. Let R be a semiperfect ring such that its Jacobson radical $J(R) = J$ is nilpotent and the quotient $R/J = K_1 \oplus \ldots \oplus K_l$ is the direct sum of fields $K_i (i = 1, \ldots l)$ and the adjoint group R° is an Engel group. If all fields K_i are algebraic over their simple subfields then

(i) $[R^\circ, R^\circ] \leq J^\circ$;

(ii) $[(J^m)^\circ, R^\circ] \leq (J^{m+1})^\circ$;

(iii) $[J^\circ, {}_m R^\circ] \leq (J^{m+1})^\circ$.

For any ring R, $R^{(2)}$ is the ideal generated by all $(r, s) = rs - sr$ with r, s in R, and inductively, $R^{(n)}$ is the ideal generated by all (r, s) with $r \in R^{(n-1)}, s \in R$. The ring R is said to be strongly Lie nilpotent if $R^{(m)} = 0$ for some m.]

In view of several known results, namely, the result of Gupta and Levin who have proved that the unit group of a Lie nilpotent ring is nilpotent of at most the same class, and the results that some Lie identities in a ring R imply the corresponding group-commutator laws in the adjoint group R°, it is natural to ask whether an Engel condition of the adjoint group R° influence Lie nilpotency of a ring R.

Proposition 2. Let R be a semiperfect ring satisfying the conditions of Proposition 1, then R° is a nilpotent group and R is a strongly Lie nilpotent ring.

Problem 1. Is the Artinian ring R with the Engel adjoint group R° Lie nilpotent?

Problem 2. Is the perfect ring R with the Engel adjoint group R° Lie nilpotent?

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