

On lower bound of growth for Gupta-Sidki's p -groups

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ABSTRACT

For any finitely generated infinite group G the growth function is defined as

$$\gamma_{G,S}(n) = |\{g; l_S(g) \leq n\}|,$$

where $l_S(g)$ is length of g in some finite system of generators S of this group. Let's say (see [1]) that a residually finite group G has *intermediate growth* if

$$e^{\sqrt{n}} \preceq \gamma_G(n) \prec e^n.$$

Nowadays, just few examples of such groups are known. First example was built by R. I. Grigorchuk [2] in 1984. Same finitely generated torsion groups are Gupta-Sidki's p -groups, p -odd [3]. It is known [4], that

$$\lim_{n \rightarrow \infty} \sqrt[n]{\gamma_{G_p}(n)} = 1,$$

so G_p has intermediate growth.

Here we give some estimation for lower bound of growth of $\gamma_{G_p}(n)$.

THEOREM. *Let p be and odd prime. For any $\varepsilon > 0$ the inequality*

$$\gamma_{G_p}(n) \succeq e^{n^{\log_{2p} p - \varepsilon}}$$

holds.

References

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- [4] J. Fabrykowski, N. Gupta, On groups with sub-exponential growth functions II, J. of the Indian Math. Soc., 56(1991), 217–228.

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