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## Generalized reflection groups

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**ABSTRACT**

Let $(G, X)$ be a pair consisting of a group $G$ and a generating set $X$ of $G$. I suppose that $X$ is a finite set and that the product of two elements of $X$ is finite. Let $K$ be a commutative ring and let $M$ be a $K[G]$-module. In this work I try to answer the following question: Is it possible to find a $K$-algebra $L$ such that $M$ is a $L[G]$-module and such that the elements of $X$ act as reflections of $M$ (which means here that $[M, x]$ is generated by one element)? For groups $G$ generated by reflections, for each reflection $x$ of $G$, $[M, x]$ is a one dimensional vector space so it is easy to see that we have the following two properties:

- Let $x$ and $y$ be two distinct commuting reflections, then $[M, x] \subset C_M(y)$;

- Let $x$ and $y$ be two non commuting reflections, then the map $\sigma_{xy} : [M, x] \to [M, y] : u \longmapsto -u + y(u)$ is a bijection.

I take these two properties as axioms:

Let $(G, X)$ have the same meanning as above. Let $K$ be a commutative ring and let $M$ be a $K[G]$-module. We introduce the following two axioms:
(Axiom 1) If $x, y \in X, xy = yx * 1$, then $[M, x] \subset C_M(y)$;
(Axiom 2) If $x, y \in X, [x, y] * 1$, then the map $\sigma_{xy} : [M, x] \to [M, y] : u \longmapsto -u + y(u)$ is a bijection.
We say then that $(G, X)$ is a generalized reflection group on $M$.

If the axioms 1 and 2 are satisfied, I show that there exists a $K$-algebra $L$ wcich acts canonically on each $[M, x],\ x \in X$. With two tecnichal further conditions, I can show that $M$ becomes an $L[G]$-module.

Now I suppose that each $x$ in $X$ is of order 2 so two distinct elements of $X$ generated a dihedral group. I study the representations of dihedral groups as generalized reflection groups which permit me to construct the algebra $L$ obtained in abstract form by generators and relations. I then have to choose a representation of $M$ which a cyclic $L$-module so that each $x$ in $X$ acts on $M$ as a reflection.

I finish by giving some examples.

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