

Infinite dimensional irreducible Lie algebras containing transformations of finite rank

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ABSTRACT

Let F be a field of characteristic zero and let V be an infinite dimensional vector space over F . A linear transformation x of V is called *finitary* if $\dim xV < \infty$.

We describe irreducible Lie subalgebras of $gl(V)$ containing nonzero finitary transformations. It turns out that any such algebra is a semidirect product of a finite dimensional Lie algebra and a “dense” Lie subalgebra of $gl(W)$ for some vector space W .

The proof is based on author’s recent classification of finitary simple Lie algebras over fields of zero characteristic [1].

This result is a Lie algebra analog of classical Nathan Jacobson’s [2, Ch.IX] description of the structure of irreducible associative rings containing finitary transformations, and Helmut Wielandt’s [3] theorem saying that each primitive permutation group on an infinite set Ω which contains a nonidentity finitary permutation contains the (finitary) alternating group $\text{Alt}(\Omega)$.

References

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