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RIT-algebra as an example of infinite dimensional Lie algebra with Noetherian universal enveloping

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ABSTRACT

We consider the question about equivalence of noetherianity of universal enveloping algebra $U(L)$ of Lie algebra L, and the finiteness of the dimension of L. It is a well-known fact that $U(L)$ for a finite-dimensional Lie algebra L is noetherian. It was shown in [1] that the opposite is true for some classes of Lie algebras. Namely, it follows from noetherianity of $U(L)$ that L is finite-dimensional for the solvable Lie algebras, for nil algebras (in characteristic 0), for algebras from a variety generated by special Lie algebra, for algebras with the Capelly identities, etc.

We show, that in general it is not true. There exists also the conjecture, that if universal enveloping algebra is noetherian and finitely presented, then L is finite dimensional. Our example shows, that it is also not true.

We consider the Lie algebra, which appears in physics in [2], [3], it is so-called Relativistic Internal Time (RIT)-Algebra. The RIT-algebra L is defined by specifying a set E_0, whose linear span coincides with L: $E_0 = \{P_\mu, J_\alpha, N_\alpha, T, V_\alpha, Q_\alpha = TV_\alpha, [V_\alpha]\}$, where $\mu = 0, 1, 2, 3, \alpha = 1, 2, 3$, and $[V_\alpha]$ — the set of all commutative monomials in variables V_α. Then L is given by the tensor γ_{ij}^k of structural constants: $[e_i, e_j] = \sum \gamma_{ij}^k e_k, e_i \in E$.

It is easy to see, that the universal enveloping algebra of $U(RIT)$ can be given by the following way: $U(L) = k\langle E\rangle / I$, where $E = \{P_\mu, J_\alpha, N_\alpha, T, V_\alpha | \mu = 0, 1, 2, 3, \alpha = 1, 2, 3\}$, and the ideal I is finitely generated.

Theorem. *RIT-algebra L is an infinite-dimensional Lie algebra with noetherian universal enveloping algebra $U(L)$.*

Let us note that we use the results about algebras of solvable type from [4].

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