Section 02: Algebra. Number Theory — Poster number 639

Extremal Betti numbers of monomial ideals

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ABSTRACT

Let K be a field and let $R = K[X_1, \ldots, X_n]$ be the polynomial ring in n variables. If $I \subset R$ is a graded ideal we define the graded Betti numbers of R/I as $\beta_{ij} = \beta_{ij}(R/I) = \dim_K \operatorname{Tor}_i(R/I, K)_j$.

A Betti number $\beta_{kk+\ell} \neq 0$ is called extremal if $\beta_{ii+j} = 0$ for all $i \geq k$ and for all $j > \ell$. One of the extremal Betti numbers computes the regularity of R/I and in this sense extremal Betti numbers can be seen as a refinement of the notion of Mumford-Castelnuovo regularity.

We examine the extremal Betti numbers of a lexsegment ideal $I \subset R$. We determine their maximum number and we give a precise characterization of the possible sequences of extremal Betti numbers for a such ideal.

Moreover, if $u_1, \ldots, u_r \in R$ are the Borel generators of an ideal $I \subset R$, that is I is the smallest strongly stable ideal which contains $u_1, \ldots, u_r$, we state conditions for which such monomials determine extremal Betti numbers of I.

Finally, we state the maximum number of such extremal Betti numbers for a strongly stable ideal of R.

References

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