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Algebraic curves defined over a finite field with many rational points

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ABSTRACT

In the current study of algebraic geometry in positive characteristic there is a growing interest to (projective, geometrically irreducible, non–singular algebraic) curves which are defined over a finite field $\mathbf{F}$ and have many $\mathbf{F}$-rational points. Such curves, especially $\mathbf{F}$-*maximal curves*, play indeed a very important role in Coding theory, and some further motivation for their investigation also comes from Number Theory and Finite Geometry. Here, maximality of a curve means that the number of its $\mathbf{F}$-rational points attains the Hasse-Weil upper bound (for $\mathbf{F} = \mathbf{F}_{q^2}$ a finite field of square order). A well–known $\mathbf{F}_{q^2}$-maximal curve is the Hermitian curve $\mathcal{H}$ of equation $X_0^{q+1} + X_1^{q+1} + X_2^{q+1} = 0$. By a result of Lachaud, any non-singular curve $\mathbf{F}_{q^2}$-covered by the Hermitian curve is also maximal. Lachaud's result pointed out two fundamental (and still open) problems, namely the classification problem of maximal curves covered by $\mathcal{H}$, and the existence problem of maximal curves which cannot be obtained in this way. [1] provides a complete classification of Galois $\mathbf{F}_{q^2}$-coverings of $\mathcal{H}$ of prime degree which together with current work by M.Abdon, A.Cossidente, R.Fuhrmann, A.Garcia, G.van der Geer, J.W.P.Hirschfeld, H.Stichtenoth, C.P.Xing plays a role in the study of maximal curves with large genera. Another recent result [2] is a characterization of maximal curves as the only curves having a $\mathbf{F}_{q^2}$-birational model $\mathcal{Y}$ embedded in a projective space $P^M(\overline{\mathbf{F}}_{q^2})$ such that $\mathcal{Y}$ lies on the Hermitian variety $X_0^{q+1} + \ldots + X_M^{q+1} = 0$ of $P^M(\overline{\mathbf{F}}_{q^2})$. This result for $M = 3$ seems to be an essential tool to the classification project of $\mathbf{F}_{q^2}$-maximal curves $\mathcal{X}$ such that dim $|(q+1)P| = 3$ for $P \in \mathcal{X}(\mathbf{F}_{q^2})$.

References

[1] A. Cossidente, G. Korchmaros and F. Torres, *On curves of large genus covered by the Hermitian curve*, Communication in Algebra, to appear.

[2] G. Korchmaros, F. Torres, *Embedding of a maximal curve in a Hermitian variety*, preprint, alg-geo/99-11-043

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