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The Poincaré lemma in crystalline cohomology of higher level

Bernard Le Stum, IRMAR, Université de Rennes I, Campus de Beaulieu, F-35042 Rennes Cedex, France.
Adolfo Quirós*, Departamento de Matemáticas, Universidad Autónoma de Madrid, E-28049 Madrid, Spain.

ABSTRACT

When working in characteristic $p > 0$, crystalline cohomology fills the gap at p in the family of étale ℓ-adic cohomologies, but it has problems. They may be dealt with using rigid cohomology, which can be seen as a limit of crystalline cohomologies of level $m \geq 0$, $m = 0$ corresponding to classical crystalline cohomology.

The theory of crystalline cohomology generalizes well to higher levels [1, 2], but the main computational tool, the de Rham complex, can not be used for $m > 0$. Following a suggestion of Berthelot [1] we have shown [3] that the "jet complex" may be used as a de Rham complex of higher level.

Theorem: *The crystalline cohomology of level m of a scheme X smooth over a p-adic base S agrees with the cohomology of a complex with locally free terms, the de Rham complex of level m, $\Omega^{\cdot}_{X/S,m}$. (We also prove versions with coefficients in an m-crystal and without the smoothness hypothesis.)*

The key ingredient is, as for $m = 0$, a formal Poincaré lemma, and the main difficulty comes from $\Omega^{\cdot}_{X/S,m}$ being much larger than expected: it is in general an infinite complex and the dimension of the terms grows with m. (For example, $dim(\Omega^1_{\mathbb{A}^1,m}) = p^m$ and $dim(\Omega^2_{\mathbb{A}^1,m}) = p^m(p^m - 1)$.) As a consequence, it is not obvious how to integrate a differential form, and checking that $\Omega^{\cdot}_{X/S,m}$ has locally free terms and is a resolution of $\mathcal{O}_X$ requires some effort.

As an application, we define the Spencer complex of higher level of X/S and show that the cohomologies of a $\mathcal{D}^{(m)}_{X/S}$-module and of its associated m-crystal coincide. Using Berthelot's Frobenius descent, we give an explicit quasi-isomorphism between the de Rham complex of level m of an m-crystal and the de Rham complex of level $m + 1$ of its Frobenius pull-back. Future applications include the study of transversal m-crystals and of the Hodge filtration on Dieudonne modules of level m.

References

[1] P. Berthelot. Letter to L. Illusie (1990).

[2] J.-Y. Etesse, B. Le Stum. Fonctions L associés aux F-isocristaux surconvergents. II. Zéros et pôles unités, *Inv. Math* **127**, 1-31 (1997).

[3] B. Le Stum, A. Quirós. The exact Poincaré lemma in crystalline cohomology of higher level, *J. of Algebra*, to appear.

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Contact Address: adolfo.quiros@uam.es