Grassmannians via projection operators and some of their special submanifolds

Ivko Dimitric, Pennsylvania State University, USA.

ABSTRACT

We consider imbeddings of real, complex and quaternion Grassmannians $Gr_k(\mathbf{F^m})$ into suitable Euclidean spaces of matrices by identifying a $k-$dimensional $\mathbf{F}-$plane ($\mathbf{F}$ being one of the fields $\mathbf{R}$, $\mathbf{C}$, $\mathbf{H}$) with the projection operator onto it. This map realizes a Grassmannian as a submanifold of a certain hypersphere of Euclidean space and this approach leads to a geometric study of various classes of submanifolds of $Gr_k(\mathbf{F^m})$. In particular, we give a characterization and some classification results for $1-$type submanifolds of $Gr_k(\mathbf{F^m})$, i.e. for those submanifolds whose position vector is (up to a translation) an eigenvector of the Laplacian on a submanifold. For rank-1 Grassmannians (projective spaces) this classification is most complete, the $1-$type submanifolds including totally geodesic projective spaces of lower dimension, minimal Lagrangian submanifolds and maximal stable geodesic hyperspheres. Several upper bounds on the first nonzero eigenvalue of the Laplacian on a compact submanifold of $Gr_k(\mathbf{F^m})$ are also obtained.

Keywords: *Grassmannian, submanifold, Laplacian, Hermitian matrices, 1-type immersion*

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Contact Address: `ivkod@alas.matf.bg.ac.yu`