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Torus actions, simple polytopes, and coordinate subspace arrangements

Victor M. Buchstaber, Department of Mathematics and Mechanics, Moscow State University.
Taras E. Panov*, Department of Mathematics and Mechanics, Moscow State University.

ABSTRACT

By means of certain manifolds defined by simple polytopes and their generalizations, the so-called moment-angle complexes, we discover and investigate new relations between topology of manifolds, combinatorics of polytopes and subspace arrangements. We show that the cohomology algebra of the complement of a coordinate subspace arrangement in m-dimensional complex space is isomorphic to the bigraded cohomology algebra of Stanley-Reisner face ring of a certain simplicial complex on m vertices. (The face ring is viewed as a module over the polynomial ring on m generators.) Then we calculate the latter cohomology algebra by means of the standard Koszul resolution of polynomial ring. To prove these facts we construct an equivariant with respect to the torus action homotopy equivalence between the complement of a coordinate subspace arrangement and the moment-angle complex defined by the simplicial complex. The moment-angle complex is a certain subset of a unit poly-disk in m-dimensional complex space invariant with respect to the action of an m-dimensional torus. This complex is a smooth manifold provided that the simplicial complex is a simplicial sphere, but otherwise has more complicated structure. In the case when the simplicial complex is the dual to the boundary complex of a simple polytope P^n the above homotopy equivalence between the complement of the arrangement and the moment-angle complex can be interpreted as the orbit map for a free action of the group $\mathbf{R}^{m-n}$ on the complement of the arrangement. The quotient (i.e. the moment-angle complex) is a smooth manifold $\mathcal{Z}_P$ with an action of torus T^m and orbit space P^n. The bigraded algebra structure in the cohomology of $\mathcal{Z}_P$ with bigraded Poincare duality enables to catch combinatorial invariants of the polytope.

Keywords: *torus action, moment-angle complex, simple polytope, subspace arrangement*

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Contact Address: tpanov@mech.math.msu.su