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A categorical description of shape through a sequence of critical points

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ABSTRACT

The poster introduces a functor as a new shape descriptor. This is a categorical evolution of size functions, descriptors already applied in several pattern recognition areas [1]. The leading ideas are: 1) "shape" is not just an object, but also the behaviour of some (continuous real) functions on it; 2) shape comparison is best performed if the "story" of the functions — at the various levels — is condensed in a formal, comparable construction. For size functions, the construction reflected the ranks of the images of the H_0–homology morphisms induced by inclusion [2]. Here a much wider setting is proposed, through extensive use of Morse theory.

Let **Morse** denote the subcategory of **Diff/R** whose objects are pairs (M, f) where M is a compact, connected manifold, and f is a Morse function on M, injective on the set of critical points, and whose morphisms are maps g between manifolds, commuting with the respecting Morse functions. Let **ES** be the category of exact sequences of abelian groups.

We define a functor $S : \mathbf{Morse} \to \mathbf{Funct}(\mathbf{Rord}, \mathbf{ES})$. Given a Morse function f on a manifold M, for each $x \in \mathbf{R}$ let $x_0, x_1 \in \mathbf{R}$ be the two critical values of f such that $x_0 < x_1 \leq x$, and there are no other critical values in the interval $[x_0, x_1]$. It is well-known that both M_x and M_{x_1} have the homotopy type of the attachment space of M_{x_0} with a λ-cell, where λ is the index of the critical point. The functor $F = S((M, f))$ is defined to associate to each x (values x below the second lowest critical value are treated aside) the Mayer–Vietoris sequence corresponding to the attachment space just described. On a morphism g its value is the induced homology morphism $H(g)$.

The functor S portraits the development of the manifold through its critical points; moreover, it keeps explicitly into account the topology of the various M_x. This is what we consider to be essential for studying the "shape" of (M, f). The presentation of each $H_k(M_x)$ can be reconstructed through the sequence of critical values, by applying the Mayer–Vietoris theorem to each cell attachment. Each critical point contributes with either a generator or a relator [3].

We envisage, as a development, the definition of distances between images $S((M, f))$ in order to allow classifications, queries and other Pattern Recognition tasks.

References

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