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Holonomy of KZ connections and Kontsevich-Vassiliev invariant

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ABSTRACT

Let $R \subset \mathbb{R}^n$ be irreducible a root system and $\nabla_R = d - \frac{1}{2}\sum_{\alpha\in R} t_\alpha d\log(\alpha, z)$ is a generalized KZ connection on $\mathbb{C}^n$ associated with R [1,2,3]. Holonomy $H_R(\gamma)$ of the ∇_R along a path γ can be represented by the Chen iterated integral series [3]. Let $Ch_n(R)$ be the chord diagram algebra of the type $R = A$, B [2,3]. Let $\Delta_i^{alg}(R)(D)$ and $\Delta_i^{geom}(R)(\gamma)$ be the doubling operations of the i-th string of a chord diagram D and of the i-th string of the graph path γ [1-3]. By definition the holonomy $H_R(\gamma)$ along *the elementary paths* γ is equal to R-matrices or Drienfeld's associators of the type R [3]. We set $t_\alpha \in Ch_n(R)$.

Theorem 1 *For arbitrary elementary path γ we have the equality*

$$H_R(\Delta_i^{geom}(R)(\gamma)) = \Delta_i^{alg}(R)(H_R(\gamma)), \qquad (*)$$

where $R = A$ or B.

Using the holonomy H_A of KZ connections and relations (*) Bar-Natan [2] have constructed the functor Z^f from the framed tangle category to the chord diagram category. The restriction of this invariant on framed links is universal Kontsevich-Vassiliev invariant. The Bar-Natan construction we realize in case of the B-type root systems using the holonomy H_B of KZ connections of the type B and corresponding relations (*). We obtain the symmetric chord diagram invariant of framed links that are fixed under rotation of the order two [3].

References

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