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On universal topological groups

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ABSTRACT

According to the Banach–Mazur theorem any separable normed space is isometrically isomorphic to a linear subspace of the Banach space $C[0,1]$. Steinhaus theorem states that any separable Banach space is isometrically isomorphic to a quotient of l_1. Analogues (or proofs of the absence of such analogues) of these theorems for different classes of topological vector spaces can be found in [Kal]. We deal with similar properties of classes of topological groups.

Topological group G is called *universal* for a class $\mathcal{K}$ of topological groups iff $G \in \mathcal{K}$ and for any $H \in \mathcal{K}$ there exists a subgroup K of G such that topological groups K and H are isomorphic.

Topological group G is called *co-universal* for a class $\mathcal{K}$ of topological groups iff $G \in \mathcal{K}$ and for any $H \in \mathcal{K}$ there exists a closed normal subgroup K of G such that topological groups G/K and H are isomorphic.

V. V. Uspenskii [Usp] proved that there exists a universal element for the class of all separable metrizable topological groups.

Theorem 1. There exists a topological group universal for the class of separable metrizable topological abelian groups.

Theorem 2. There exists a topological group co-universal for the class of separable metrizable complete topological abelian groups.

Theorem 3. There exists a topological group co-universal for the class of separable metrizable Raikov-complete topological groups.

Note that co-universal elements for classes of all separable metrizable topological groups and separable metrizable topological abelian groups do not exist.

References

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