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On the Kontsevich integral

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ABSTRACT

The Kontsevich integral was constructed in 1992 but just in 1997 D.Bar-Natan obtained explicit formula for the trivial knot. To find analogus formula for an arbitrary knot is an open problem. We discuss a formula for computing degree 2 and 3 terms of the Kontsevich integral for arbitrary knots.

Let K be an oriented knot and K_m^{sing} be a singular knot with m double points. By definition Vassiliev's degree n invariant V_n is invariant vanishing on K_m^{sing} for $\forall m > n$. We denote by $\mathcal{D}_n$ a linear vector space over $\mathbf{Q}$ generated by chord diagrams (on the circle) with n chords. A linear vector function $W_n : \mathcal{D}_n \to \mathbf{Q}$ is called a weight degree n system if it is satisfied 1- and 4-term relations.

We consider generic planar projection of a knot with marked "base point". Any crossing x of such projection we're equipped with two "coordinates": $\delta_x \in \{0, 1\}$ and $\varepsilon_x \in \{\pm 1\}$, where ε_x is a local writh number, and δ_x is defined by the ordering of passing of over- and undercrossings (start at the "base point").

The Kontsevich integral $Z(K)$ is a series of chord diagrams with numerical coefficients. Finit degree invariant V_n is determined by W_n of these chord diagrams. The universal Vassiliev knot invariant is defined as the following modified Kontsevich integral: $I(K) = \frac{Z(K)}{Z(\cup)^{c(K)/2}}$, where $\cup$ is the trivial knot, $c(K)$ is a number of critical points of K.

Proposition. *Formula for computing of the universal Vassiliev invariant modulo 4-degree terms for an arbitrary knot K is the following:*

$$I(K) = 1 + \frac{1}{2}[\sum_{\{x,y\}} (-1)^{\delta_x+\delta_y} W_2(\{x,y\})\varepsilon_x\varepsilon_y[\delta_x(1-\delta_y) + \delta_y(1-\delta_x)] - \frac{1}{12}] \bigotimes +$$

$$\frac{1}{4}[\sum_{\{x,y,z\}} (-1)^{\delta_x+\delta_y+\delta_z} W_3(\{x,y,z\})\varepsilon_x\varepsilon_y\varepsilon_z[\delta_x(1-\delta_y)\delta_z - (1-\delta_x)\delta_y(1-\delta_z)]] \circledast,$$

where sums are taken over all pairs (resp. triplets) of double points of the knot projection.

Keywords: *knot, Vassiliev's invariants, chord diagram, weight system, Kontsevich integral*

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