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Latin Squares without Proper Subrectangles and Perfect Factorisations of Bipartite Graphs

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ABSTRACT

A Latin square is *pan-Hamiltonian* if the permutation which defines row i relative to row j consists of a single cycle, regardless of the choice of $i \neq j$. A Latin square is *atomic* if all its conjugates are pan-Hamiltonian. In other words, a square is atomic if all the row, column and symbol cycles are Hamiltonian. The simplest examples are the Cayley tables of groups of prime order.

A *perfect factorisation* of a graph is a decomposition of that graph into matchings such that the union of any two matchings is a Hamiltonian cycle. Each pan-Hamiltonian Latin square of order n describes a perfect factorisation of the complete bipartite graph $K_{n,n}$, and vice versa.

I will present various existence results and conjectures for pan-Hamiltonian and atomic Latin squares, plus results of enumerations for small orders. I will also explore possible uses of these squares and some of their interesting properties. For example, these squares are all free of proper subsquares, in fact they have no non-trivial Latin subrectangles.

Keywords: *Latin square, perfect factorisation, bipartite graph*

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