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Clustering and isoperimetric properties of weighted graphs via Laplacian eigenvalues and eigenvectors

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ABSTRACT

In [1], the Laplacian eigenvalues of a weighted graph were investigated together with k-dimensional Euclidean representatives of the vertices (k is a given integer between 1 and the number of vertices). Their relation to the minimal k-cut of the weighted graph was also established.

Now, we would like to classify the vertices into k clusters in such a way that vertices having "many small-weight" edges in common belong to different clusters, but in this case even cluster-volumes are preferred (the volume of a cluster is the sum of the weights of its vertices, where consistent weighting is used). For this purpose the notion of weighted k-density is introduced that is a generalization of the minimal k-cut in [1] and that of the isoperimetric number and Cheeger-constant discussed by B. Mohar and F. Chung in [3] and [2]. Its relation to the k smallest positive eigenvalues of the weighted Laplacian is investigated. In fact, we have a quadratic objective function, and the weighting influences only the conditions imposed on the representatives. The representation plays an important role in the proof and it also helps us in finding the clusters themselves.

If the sum of the weights is 1 (this is not a restriction), then the problem can be treated in terms of joint probability distributions, conditional expectation and the eigenvalues are related to the canonical correlations. Hence, the above classification problem is closely related to the correspondence analysis on the weight matrix.

References

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