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Symmetric block designs (71,15,3) admitting an automorphism of order 6

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ABSTRACT

We proved the following

THEOREM. There are, up to isomorphism, 146 symmetric block designs (71,15,3) admitting an automorphism of order 6. Among them 68 pairs are dual. The full automorphism groups of these designs are isomorphic to $S_3 \times E_4, A_4 \times Z_2, E_4 \times A_4, E_8 \cdot F_{21}, (E_8 \cdot F_{21}) \times Z_2$.

Designs (71,15,3) are the greatest known designs with parameters $(v, k, 3)$. The obtained results include all the 11 previously known $(71, 15, 3)$–designs (see [2],[3]).

For a (v, k, λ)–design $\mathcal{D} = (\mathcal{P}, \mathcal{B}, I)$ with point set $\mathcal{P}$, line set $\mathcal{B}$ and incidence I, denote by $\langle x \rangle$ the set of points incident with line x, and by $\langle P \rangle$ the set of lines incident with point P. Let G be an automorphism group of $\mathcal{D}$, and $\mathcal{B}_i, \mathcal{P}_r, 1 \leq i, r \leq t$ the G-orbits of lines and points, respectively. For $x \in \mathcal{B}_i, P \in \mathcal{P}_r$ the cardinalities $\gamma_{ir} =\mid \langle x \rangle \cap \mathcal{P}_r \mid$, $\Gamma_{ir} =\mid \langle P \rangle \cap \mathcal{B}_i \mid$ do not depend on the choice of x and P. They satisfy some important relations, essential for the construction of designs (see [1]).

Our construction was carried out in three steps:

1) Construction of orbital structures, $t \times t$–matrices (γ_{ir}), for $\mathcal{D}$ with respect to the assumed automorphism ρ.

2) Partial indexing of orbital structures with respect to the factor group $\langle \rho \rangle / \langle \rho^3 \rangle$, that is, constructing new orbital structures with respect to $\langle \rho^3 \rangle$, taking account of the action of $\langle \rho \rangle$ on the $\langle \rho^3 \rangle$–orbits.

3) Final indexing of partially indexed structures — construction of design incidence matrices.

References

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