

Local transformations of combinatorial objects in the plane

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ABSTRACT

Given a set P_n of n points in the plane, a *non-crossing* graph on P_n is a graph having P_n as vertex set and whose edges are straight line segments that do not cross. Given a family \mathcal{G} of non-crossing graphs on P_n and a local rule for transforming an object of \mathcal{G} into another object of \mathcal{G} by exchanging a (usually small) set of edges, define the *flip-graph* G associated to \mathcal{G} as follows. G has \mathcal{G} as vertex set, and edges join pairs of objects in \mathcal{G} if they can be transformed into each other by means of a single application of the transformation rule.

In recent years we have studied flip-graphs extensively for the family of triangulations of a point set [5,6], spanning trees [2,3], perfect matchings [4], and simple polygons [1]. Our results are concerned mainly with properties of distances in flip-graphs, connectivity, symmetry and Hamilton cycles.

Besides its intrinsic combinatorial interest, we remark that flips-graphs have been widely used in enumeration, random generation and optimization problems.

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