

Stability of a Partitioning Algorithm for Special Classes of Banded Linear Systems

Velisar Todorov Pavlov, Center of Applied Mathematics and Informatics, University of Rousse, Rousse, Bulgaria.

ABSTRACT

Banded systems of linear equations we can solve in parallel by so called partition methods. A typical member of these methods in the case of tridiagonal systems is the method of Wang [1]. This method gives an efficient parallel algorithm for solving such systems. Full roundoff error analysis for the whole algorithm in the case of nonsingular tridiagonal matrices is presented in [2].

A generalized version of this parallel partitioning algorithm later has been applied from the other authors. Backward componentwise error analysis of this generalized version can be found in [3]. In this work are obtained bound on the equivalent perturbations depending on three constants and then are presented bound on the forward error as well depending on two types of condition numbers.

In the present work we consider more precisely the case when matrix A of the system belongs to one of the following classes: diagonally dominant, symmetric positive definite, or M -matrices. We prove that the algorithm is numerically stable for the considered three classes of matrices.

Unfortunately when the matrix of the system does not belong to the above mention classes, the algorithm can break down or behaves poorly. In our paper we present also a stabilization version of the generalized Wang's algorithm for banded linear systems. In brief, we stabilize the algorithm by means of perturbing the results. In consequence, we get a solution which is perturbed, and it close to the exact solution. Then, we apply iterative improvement to obtain a more accurate solution. In practice we need one iteration to get very accurate solutions.

The outline of the paper is as follows. Section 1 presents introduction and describes the Wang's partitioning algorithm for banded systems. In Section 2 we consider special classes of matrices. In Section 3 we present a stabilization version of the algorithm. Finally, some numerical experiments in MATLAB are reported in Section 4.

References

- [1] H. Wang, A parallel method for tridiagonal linear systems, *ACM TOMS*, 7 (1981), pp. 170–183.
- [2] P. Yalamov and V. Pavlov, Stability of a partitioning algorithm for tridiagonal systems, *SIAM J. Matrix Anal. Appl.*, v. 20, N 1 (1999), pp. 159–181.
- [3] P. Yalamov and V. Pavlov, Backward Stability of a Parallel Partitioning Algorithm for Banded Linear Systems, *Proc. of 4th International Conference on Numerical Methods and Applications*, (Eds. O. Iliev et. al.), Sofia, August 19–23, 1998, World Scientific Publ., 655–663, 1999.

Keywords: *partitioning method, banded systems, roundoff error analysis*

Mathematics Subject Classification: *65G05, 65F05, 65Y05*

Contact Address: *velisar@ami.ru.acad.bg*