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Two-dimensional wave packets in a generalized elastic solid

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ABSTRACT

The Davey-Stewartson (DS) system was introduced in [1] as a model for the evolution of weakly nonlinear packets of water waves that travel in one direction but in which the amplitude of waves is modulated in both spatial directions. Thus, the resulting system for the complex amplitude $A(\xi,\eta,\tau)$ and the real potential $\phi(\xi,\eta,\tau)$ is written as

$$\begin{aligned} &iA_\tau + \lambda A_{\xi\xi} + \mu A_{\eta\eta} = \nu|A|^2 A + \nu_1 A\phi_\xi \\ &\alpha\phi_{\xi\xi} + \phi_{\eta\eta} = -\beta(|A|^2)_\xi \end{aligned} \tag{1}$$

where the real coefficients in (1) depend on the physical parameters of the problem.

The object of the present study is to investigate the propagation of two-dimensional wave packets in a weakly nonlinear and weakly dispersive elastic solid. To this end, a multi-scale expansion is used to find the evolution of 2D wave packets propagating in an infinite nonlinear elastic medium with higher order gradients [2]. The following coupled nonlinear evolution equations are obtained for the description of the propagation of the short wave amplitude u_3 and the long waves u_1 and u_2

$$\begin{aligned} &\lambda_1\frac{\partial^2 u_1}{\partial\xi^2} + \lambda_2\frac{\partial^2 u_1}{\partial\eta^2} + \lambda_3\frac{\partial^2 u_2}{\partial\xi\partial\eta} = a\frac{\partial}{\partial\xi}|u_3|^2, \\ &\mu_1\frac{\partial^2 u_2}{\partial\xi^2} + \mu_2\frac{\partial^2 u_2}{\partial\eta^2} + \lambda_3\frac{\partial^2 u_1}{\partial\xi\partial\eta} = b\frac{\partial}{\partial\eta}|u_3|^2, \\ &i\,\frac{\partial u_3}{\partial\tau} + c\,\frac{\partial^2 u_3}{\partial\xi^2} + d\,\frac{\partial^2 u_3}{\partial\eta^2} + e\,|u_3|^2 u_3 = a u_3\frac{\partial u_1}{\partial\xi} + b u_3\frac{\partial u_2}{\partial\eta}, \end{aligned} \tag{2}$$

where the coefficients are real and depend on the material parameters. Since the field equations of the generalized nonlinear elastic medium are highly coupled in two-dimensions, the system (2) which describes the behavior of two-dimensional wave packets, has a more general form than the DS system.

The stability of uniform wavetrains are also discussed. The stability criterion given by Hashimoto and Ono [3], and generalized by Davey and Stewartson to the case of two-dimensional waves, is used to investigate the modulational instability of wave trains. The instability regions which depends mainly on the material parameters are also presented.

References

1. A Davey and K Stewartson, *Proc. R. Soc Lond. A* **338** 101-110 (1974).
2. VI Erofeyev and AI Potapov, *Int. J. Non-Linear Mechanics* **28**, 483-488 (1993).
3. H Hashimoto and H Ono, *J. Phys. Soc. Japan* **33**, 805-811 (1972).

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