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Second-Order Necessary Conditions for Hyperbolic Differential Inclusion Problems

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ABSTRACT

Consider the following problem

$$\text{minimize } g(u(T_1, T_2)) \tag{1}$$

over the solutions of the hyperbolic differential inclusion

$$u_{xy}(x,y) \in F(x,y,u(x,y)), \quad (x,y) \in [0,T_1] \times [0,T_2] \tag{2}$$

with boundary conditions

$$\begin{array}{l} u_x(x,0) \in F_1(x,u(x,0)), \quad x \in [0,T_1] \\ u_y(0,y) \in F_2(y,u(0,y)), \quad y \in [0,T_2] \\ u(0,0) \in X_0 \end{array} \tag{3}$$

and satisfying end point constraints

$$u(T_1,T_2) \in X_1 \tag{4}$$

This optimization problem has been extensively studied in the literature, mainly, when F has a parametrized form, i.e. controlled hyperbolic differential equations. First-order necessary conditions for the problem (1)-(4) are well known.

We obtain second-order necessary optimality conditions for the problem (1)-(4) by reducing the (infinite-dimensional) optimal control problem to the finite-dimensional problem of minimizing the terminal payoff on the intersection of the (known) target set with the (unknown) reachable set and to use a general result from nonsmooth analysis. Let us mention that this approach has been, already used to obtain second-order necessary optimality conditions for problems given by “ordinary” differential inclusions.

Keywords: *Optimal control, Hyperbolic differential inclusion, Second-order necessary condition, Second variation*

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