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On a Certain Boundary Value Problem for Nonlinear Ordinary Differential Equations

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ABSTRACT

The nonlinear ordinary differential equation of the type

$$u^{(n)} + \sum_{k=1}^{n-1} p_k(t) u^{(k)} = f(t, u, u^{'}, ..., u^{(k)}) \tag{1}$$

is considered on $[a, +\infty[$, where each of the function $p_k : [a, +\infty[\to \mathbf{R}$ for $(k = 1, ..., n-1)$ is locally absolutely continuous together with its derivatives up to order $k-1$ inclusive and the function $f : [a, +\infty[\times \mathbf{R^n} \to \mathbf{R}$ satisfies the local Carathéodory conditions. For equation (1) the nonlinear boundary value problem

$$\begin{gathered} u^{(i)}(a) = \varphi_i(u(a), u'(a), ..., u^{(n-1)}(a)) \ \ (i = 0, ..., n_0 - 1), \\ \int_a^{+\infty} |u^{(j)}(t)|^2 dt < +\infty \ \ (j = 0, ..., n_0) \end{gathered} \tag{2}$$

and some modifications of it are studied, where n_0 is the entire part of the number $\frac{n}{2}$, while the functions φ_i $: \mathbf{R^n} \to \mathbf{R}$ $(i = 0, ..., n_0 - 1)$ are continuous.

The results are new even for Emden-Fowler type equation

$$u^{(n)} + u^{(n-2)} = p(t)|u|^{\lambda} sgn\ u, \tag{3}$$

and they answer some open problems of oscillatory theory. Namely, It was known (see I. Kiguradze [1]) that if $n \geq 4$ is even, $\lambda > 1$ and $p : [0, +\infty[\to] - \infty, 0]$ is locally summable, then the condition

$$\int_0^{+\infty} t^{n-3} p(t) dt = -\infty$$

is necessary and sufficient for every *proper solution* (i.e., nontrivial solution defined in some neighborhood of infinity) of (3) to be oscillatory. However, the question on the existence of at least one proper solution of (4) remained open. We proved that if n and n_0 are even and

$$p(t) \leq -\delta t^{n-2} \ \ for \ \ t \geq a,$$

where $\delta > 0$, then equation (3) possesses the n_0-parametric family of proper oscillatory solutions.

Reference

I. Kiguradze, An oscillatory criterion for a class of ordinary differential equations, (Russian) *Differentsial'nye Uravnenya* **28** (1992), No. 2, 207–219.

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