3rd European Congress of Mathematics
Poster sessions

Section 08: Ordinary Differential Equations and Dynamical Systems | Poster number 226

Optimal Control of Nonlinear Oscillatory System

Natalia Balashevich, Institute of Mathematics, NASB, Minsk, Belarus.

ABSTRACT

Consider a problem of optimization of a nonlinear dynamical system by bounded controls

$$c'x(t^*) \longrightarrow \max, \quad \dot{x} = f(x) + bu, \quad x(0) = x_0, \quad Hx(t^*) = g, \quad |u(t)| \le L,\ t \in T = [0, t^*], \tag{1}$$

$$\Big(x \in R^n,\ u \in R,\ g \in R^m,\ rankH = m < n\Big).$$

The idea of the proposed approach to solving problem (1) consists in introducing a piecewise linear approximation of function $f(x)$ and asymptotic optimization of a piecewise quasilinear system.

Let X be a domain of the state space of system (1) where its behaviour is investigated. Divide X into polyhedral sets $X_1, \dots, X_p$ such that $\bar{X} = \bigcup_{i=1}^{p} X_i$, $\mathrm{int}X_i \cap \mathrm{int}X_j \neq \emptyset$, $i \neq j$. Introduce a function $\hat{f}(x)$ continuous on X and linear on each set X_i, $i = \overline{1,p}$: $\hat{f}(x) = A_i x + \alpha_i$, $x \in X_i$. A value $\delta = \max_{x \in X} \|f(x) - \hat{f}(x)\|$ is said to be a degree of approximation of function $f(x)$, $x \in X$.

Problem (1) is embedded into the family of problems

$$c'x(t^*) \longrightarrow \max, \quad \dot{x} = \hat{f}(x) + \mu g(x) + bu \quad x(0) = x_0, \quad Hx(t^*) = g, \quad |u(t)| \le L,\ t \in T, \tag{2}$$

depending on a parameter μ. Here $g(x) = (f(x) - \hat{f}(x))/\delta$, $x \in X$.

At first, a base problem for family (2) is considered. It is derived from (2) when $\mu = 0$. Actually, it is a problem of optimal control for a step-linear system where instants of transition between domains of linearity are chosen together with control $u(t)$, $t \in T$.

For constructing a solution to a step-linear problem an effective algorithm based on the use of advanced linear programming technique [1] is proposed.

Then on the base of the approach [2] to optimization of quasilinear systems an asymptotic expansion of switching points of optimal control, the Lagrange multipliers and instants of transition between domains of linearity is constructed.

For a fixed value $\mu = \delta$ problem (2) coincides with problem (1) so with this value a suboptimal solution to problem (1) is obtained.

The proposed approach is illustrated by the problem of optimal damping of a nonlinear pendulum. Results of numerical experiments are discussed.

References

[1] Gabasov, R. Adaptive method of solving linear programming problems. *Preprint Series of University of Karlsruhe*, 1994.

[2] Kalinin A.I. Optimization of quasilinear control systems. Comp. Maths Math. Phys., 1988, vol. 288.

Keywords: *optimal control, piecewise linear approximation, asymptotic method*

Mathematics Subject Classification: *49M35*

Contact Address: `balash@im.bas-net.by`