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Periodical solutions of the Liénard equation with impulsive effects

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ABSTRACT

There are a lot of different phenomena in nature and technical systems in which short–time effects of shock type are occurred. Similar problems can be studied by means of application of the theory of impulsive differential equations [1]. The corresponding mathematical models are described by differential equations and additional so–called conditions of impulsive influences.

We study nonlinear differential Liénard equation

$$\ddot{x} + f(x)\dot{x} + g(x) = 0, \tag{1}$$

where $f(-x) = -f(x)$, $g(-x) = -g(x)$ and $xg(x) > 0$ for $x \neq 0$, with impulsive effects

$$\Delta \left.\frac{dx}{dt}\right|_{x=x_*} = \left.\frac{dx}{dt}\right|_{t=t_*+0} - \left.\frac{dx}{dt}\right|_{t=t_*-0} = I(\dot{x}). \tag{2}$$

The function $g(x)$ characterizes returning force and $f(x)$ is usually a varied fading decrement. The system (1) is frequently used in different applications in mechanics for describing strong interactions [1].

The impulsive effects in the system (1), (2) occur at the non–fixed moments of time [2], when the moving point passes some fixed position $x = x_*$. At these moments of time the quantity of movement in the system is increased by some value $I(\dot{x})$, depending on velocity $\dot{x}(t)$ of the point.

A function $x(t)$ is called a solution of the problem (1), (2) if it is continuous with respect to $t \in \mathbb{R}$ and continuously differentiable for all $t \in \mathbb{R}$ except the moments of impulsive effects, when its derivative $\dot{x}(t)$ is continuous from the right.

By using approach based on application of the Poincáre function and method of phase plain qualitative analysis of behaviour of phase trajectories for the problem (1), (2) had been done. In particular, sufficient conditions under which the problem (1), (2) has periodical solutions with exactly n impulses per period are obtained.

As examples, the Duffing equations and equation of mathematical pendulum are considered in details.

References

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