3rd European Congress of Mathematics
Poster sessions

Section 08: Ordinary Differential Equations and Dynamical Systems | Poster number 276

A symplectic study of motions in a perturbed Van-der-Pol dynamical system

Y. A. Prykarpatsky*, Institute of Mathematics of NAS of Ukraine.
A. M. Samoilenko, Institute of Mathematics of NAS of Ukraine.

ABSTRACT

There is studied a weakly perturbed Van-der-Pol dynamical system and the structure of its motions via the modern symplectic theory. Based on a Samoilenko-Prykarpatsky method of treating integral submanifolds of perturbed completely integrable Hamiltonian systems one proves the regular deformation of the Lagrangian asymptotic submanifolds in a vicinity of the hyperbolic periodic orbit.

We shall deal with the following Van-der-Pol dynamical system

$$dx/dt = y, \quad dy/dt = -\omega_0^2 x - \alpha y(x^2 - \beta^2) + \varepsilon y \cos \omega t, \tag{1.1}$$

where $(x, y) \in T^*(\mathbf{R})$, $\omega_0, \omega \in \mathbf{R}_+$ — the frequencies, α, β, $\varepsilon \in \mathbf{R}_+$ — some given constans and $t \in \mathbf{R}$ — the evolution parameter. There is assumed too that parameters $\alpha, \varepsilon \in \mathbf{R}_+$ are small enough.

It is evident that at $\alpha, \varepsilon = 0$ dynamical system (1.1) doesn't possess hyperbolic peculiar points or periodic curves. This fact prompts us to devise a generalization of the theory developed in [1,2,3] for treating weakly perturbed Hamiltonian systems and their irregular motions caused by possible transversal separatrix splitting via the Birkhoff-Smale scenario. Herewith we suggest a new approach based on Samoilenko-Prykarpatsky imbedding submanifolds theory , to studying such a class of problems having no hyperbolic invariant submanifolds at $\alpha, \varepsilon = 0$. In the case of the dynamical system (1.1) we prove at $\varepsilon = 0$, $\alpha \in \mathbf{R}_+$ small enough the existence of the stable hyperbolic periodic invariant curve whose stable and unstable Lagrangean submanifolds are regular at $\varepsilon \in \mathbf{R}_+$ small enough in a vicinity of this attractive invariant curve.

References

[1]. Wiggins S. On the detection and dynamical consequences of orbits homoclinic to hyperbolic invariant tori in a class of ordinary differential equations. SIAM Journal Appl.Math., 1978, 48, 262–285.

[2]. Melnikov V.K. On the center stability at periodic in time perturbations. Proceedings of the Moscow Math.Soc.,1963,12,3–52.

[3]. Samoilenko A.M., Tymchyshyn O., Prykarpatsky A.K. The geometric Poincare-Melnikov analysis of transversal splitting separatrix manifolds of slowly perturbed nonlinear dynamical systems. Ukr. Math. Journ., 1993, 45, N12, 1668–1681.

Keywords: *Van-der-Pol, dynamical system, pertubation*

Mathematics Subject Classification: *37J05, 37J15, 37J35, 37J40*

Contact Address: `yarchyk@imath.kiev.ua`