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Dynamic in small neighborhoods of hyperbolic periodic points and coding of heteroclinic trajectories.

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ABSTRACT

Objects with hyperbolic periodic points ranks high in a geometrical theory of dynamical system since owing to Grobman-Hartman theorem hyperbolic periodic points are locally structurally stable.

Consider a diffeomorphism with hyperbolic periodic points on 2-manifold, whose stable and unstable manifolds intersects each other transversally, a so called Cupka-Smale diffeomorphism. Due to hyperbolicity, every periodic point possesses a small neighborhood where stable and unstable manifolds of saddle periodic points intersects each other transversally in one point, creating a lattice. For stucturally stable diffeomorphisms (diffeomorphisms for which not only periodic points but the whole set of non-wandering points is hyperbolic and its stable and unstable manifolds intersects each other transversally) it means that the periodic point possesses a neighborhood with a local structure of direct product.

The simplest example of such a diffeomorphism is a Morse-Smale diffeomorphism which is a stucturally stable diffeomorphism with finite set of non-wandering points. In this case we have a finite number of saddle periodic points possessing small neighborhoods with lattice structure inside and can extend those neighborhoods to neighborhoods of their stable and unstable manifolds. At that there appear regular patterns in location of points of the stable and unstable manifolds intersection, connected with those lattice neighborhoods, and, introducing numerical characteristics of regularity we can describe every point of intersection and specify the location of every point of the intersection (a so called heteroclinic point) using finite amount of information. In author's paper [2] in such way there was obtained the topological classification of Morse-Smale diffeomorphisms on 2-manifolds. In that year C. Bonatti and R. Langevin obtained in paper [1], the more general result, the topological classification of structurally stable diffeomorphisms on 2-manifolds. There was used Markov topological chains to represent the diffeomorphism. Constructions similar to previously described may be used in that case also. It gives us another proof of this fact with the corresponding topological invariants and an algorythm of its construction given in an explicit form.

Implementations of this construction in higher-dimensional case and for more wide classes of diffeomorphisms than structurally stable diffeomorphisms may be also interesting.

References

[1] C. Bonatti and R. Langevin, Diffeomorphismes de Smale des surfaces. Asterisque 250 (1998), 243 pages.

[2] Igor Vlasenko, Complete invariant for two-dimensional Morse-Smale diffeomorphisms. e-Print, math.DS/9812063

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