3rd European Congress of Mathematics
Poster sessions

Section 08: Ordinary Differential Equations and Dynamical Systems | Poster number 343

Using Equational Resurgence in Hamilton-Jacobi Equation to Compute Splitting of Separatrices in the Singular Case

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ABSTRACT

It is known that the splitting of separatrices gives rise to complicated behaviour of trajectories in hamiltonian dynamical systems. Our main purpose is to compute the splitting in a rapidly forced hamiltonian system, using complex matching and resurgence methods. A typical example which presents this kind of phenomenology is the pendulum with periodic forcing:

$$H_{\mu,\varepsilon}(q,p,t) = \frac{p^2}{2} - 1 + \cos q + \mu(1-\cos q)\sin(t/\varepsilon)$$

where μ and ε are small parameters.

Stable and unstable manifolds of the periodic solution are given by $p = \frac{\partial S}{\partial q}(q,t)$, where S are t-periodic solutions of Hamilton-Jacobi equation $\partial_t S + H_{\mu,\varepsilon}(q, \partial_q S, t) = 0$.

Both manifolds are approached in the real domain by a power series in ε, called the outer series, which is the formal solution of Hamilton-Jacobi equation. This fact proves a known result which states that the distance between these manifolds is $O(\varepsilon^n)\,\forall n$. All terms in this series present a singularity at the same complex point. Following matching techniques near this singularity, Hamilton-Jacobi equation is given, in first order with respect to ε, by a partial differential equation which does not depend on ε, the so-called inner equation.

The problem is to obtain two solutions of this equation, periodic in t and asymptotic to a formal solution in powers of $1/\rho$. The behaviour of these solutions is given by the formal Borel transform of this series and its analytic continuation. Equational resurgence is used for this study and it will give us the diference between the solutions $T^{\pm}(\rho,t)$: resurgence proves that the principal component of the diference is basically given by the linearization of the inner equation.

References

[1] D. Sauzin. Résurgence paramétrique et exponentielle petitesse de l'écart des séparatrices du pendule rapidement forcé. *Ann.Ins.Fourier*, **45**(2):453-511, (1995).

[2] C. Bonet, D. Sauzin, T.M. Seara and M. València. Adiabatic invariant of the harmonic oscillator, complex matching and resurgence methods. *SIAM J. Math. Anal.*, **29**(6):1335-1360, (1998).

[3] V. Gelfreich. Reference systems for splitting of separatrices. *Nonlinearity*, **10**(1): 175-193, (1997).

[4] B. Candelpergher, J.C. Nosmas, F. Pham. Approche de la résurgence. *Actualités Math. Hermann*,(1993).

Keywords: *exponentially small, matching theory, resurgence theory*

Mathematics Subject Classification: *30B, 34E, 40C, 58F*

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