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Dependence on parameters for the Dirichlet problem

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ABSTRACT

We investigate the nonlinear Dirichlet problem:

$$\frac{d}{dt} L_{x'}(t, x'(t)) + V_x(t, x(t)) = 0, \text{ a.e. in } [\mathbf{0}, \mathbf{T}] \tag{1.1}$$

$$x(0) = 0 = x(T),$$

where

(H) $T > 0$ *is arbitrary,* $L, V : \mathbf{R} \times \mathbf{R}^n \to \mathbf{R}$ *are convex, Gateaux differentiable in the second variable and measurable in* t *functions.*

We are looking for solutions of (1.1) being a pair (x, p) of absolutely continuous functions $x, p : [0, T] \longrightarrow \mathbf{R}^n$, $x(0) = 0 = x(T)$ such that

$$\frac{d}{dt} p(t) + V_x(t, x(t)) = 0,$$
$$p(t) = L_{x'}(t, x'(t)).$$

We assume that V_x is superlinear. (1.1) is the Euler - Lagrange equation to the functional

$$J(x) = \int_0^T (-V(t, x(t)) + L(t, x'(t)))dt \tag{1.2}$$

considered on the space $A_{0,0}$ of absolutely continuous functions $x : \mathbf{R} \to \mathbf{R}^n$, $x(0) = 0 = x(T)$. (1.2) is, in general, unbounded in $A_{0,0}$ (especially in superlinear case), therefore we must look for critical points of J of "*minmax*" type. Our aim is to find a nonlinear subspace X of $A_{0,0}$ defined by the type of nonlinearity of V (and in fact also L). Taking into account the structure of the space X we shall study the functional

$$J(x) = \int_0^T (-V(t, x(t)) + L(t, x'(t)))dt$$

on the space X. We shall look for a "min" of J over the set X i.e.

$$\min_{x \in X} J(x).$$

To show that element $\bar{x} \in X$ realizing "min" is a critical point of J we develop a duality theory between J and dual to it J_D.

Keywords: *duality, variational method, superlinear differantial equations, Dirichlet problems*

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